Color Categories are Not Arbitrary

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1. Challenges to the Universals and Evolution theory of basic color term naming

A theory of basic color term meanings based on the observation of semantic universals and evolutionary regularity in the naming of colors across languages was proposed by Berlin and Kay (1969). This model has undergone numerous revisions. I will refer to the various versions collectively as the UE model. A constant feature of all versions of the UE model has been the observation that there are universal constraints on the naming of colors, that it is not the case, for example that “Our partitioning of the spectrum consists of the arbitrary imposition of a category system upon a continuous physical domain” (Krauss 1968: 268-269).

Two recent studies have added important findings in the area of cross language color naming. Levinson’s (2000) study of Yêlì Dnye[1] color naming has produced the first experimentally documented example of a language lacking a set of basic color terms that partitions the perceptual color space. These and related findings (e.g., Lyons 1995, 1999) have required revision of the UE model to take account of such Emergence Hypothesis (EH) phenomena (Kay 1999, Kay and Maffi 1999).[2] Roberson, Davies and Davidoff’s (2000) study of Berinmo[3] color terms and related psychological tests (hereafter RDD) failed to replicate some important results of Rosch’s Dani work (Heider 1972a,b, Heider and Olivier 1972), which had appeared

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[1] Yêlì Dnye is a language isolate spoken on Rossel Island in the Louisade Archipelago, Papua New Guinea.

[2] Levinson’s Yêlì Dnye results were widely available in preprint form before the published paper appeared.

[3] Berinmo is the name given by Davidoff et al. for the language of "a remote, previously unstudied, hunter-gatherer tribe,... which lives on the upper reaches of the Sepik River in Papua New Guinea" (Davidoff, Davies, and Roberson 1999: 203). No information is given by these authors regarding the genetic affiliation of this language, if any. To my knowledge, the name Berinmo did not appear in the literature prior to Davidoff *et al.* (1999).

to show a perceptual basis for the universal constraints on color naming observed by Berlin and Kay.[4] Neither of these studies, however, has produced information challenging the UE claim of universal constraints on cross language color naming *per se*. Some explicitly relativist conclusions regarding color *naming* have nevertheless been drawn. For example RDD write, "... we will propose that color categories are formed from boundary demarcation based predominantly on language. Thus in a substantial way, we will present evidence in favor of linguistic relativity" (2000: 394). More recently, Davidoff has written, "… our own cross-cultural research … indicates that perceptual categories are derived from the words in the speaker's language. The new data support a rather strong version of the Whorfian view that perceptual categories are organized by the linguistic system of our mind" (Davidoff 2001: 382). The present paper will demonstrate that not only do the color naming data of Yêlì Dnye and Berinmo fail to challenge the existence of universal constraints on color naming, these data provide strong evidence for universal constraints on color naming.[5]

2. Berinmo and Yêlì Dnye color naming

[4] And by numerous others. See, for example, the results of two large surveys reported in MacLaury (1997) and Kay et al. (1997), described below, as well as the references in these works to studies of individual languages. The bibliography added by Luisa Maffi to the 1991 paperback reprinting of Berlin and Kay (1969) also contains citations of numerous individual-language studies confirming the general picture of cross language constraints on color naming.

[5] An earlier version of the UE model (Kay and McDaniel 1978) asserted that established universals in color naming could be explained by known properties of the visual system, specifically the differential firing rates of classes of macaque LGN neurons found by De Valois et al. (1966). Although grounding the phenomenal opponency of red/green and yellow/blue in monkey LGN firing rates was widely accepted in the vision community at the time it has since been abandoned (Abramov 1997, Abramov and Gordon 1994, Derrington et al. 1984). Kay and Maffi (1999: 746) explicitly reject the hypothesis that LGN firing rates provide a biological basis for the UE model. The Kay and Maffi model takes universal constraints on color naming to be based on presumed universals of color appearance – for example, on opponent red/green and yellow/blue phenomenal channels – but on no specific neural substrate, retinal, geniculate, or cortical.

In order to test whether color naming in Berinmo and Yêlì Dnye supports the hypothesis of universal constraints on color naming, a point location in the stimulus space was calculated for each Berinmo and Yêlì Dnye chromatic color term, specifically, the centroid of the naming responses for that term.[6] MacLaury (1997: 467), on the basis of inspection of the full World Color Survey (WCS) dataset (see note 9) as well as the Mesoamerican Color Survey dataset (MacLaury 1997), isolated four chips in the stimulus array as representing the universal "elemental" hues: red, yellow, green and blue. The test of compatability of the Berinmo and Yêlì Dnye hue naming data with universal patterns of color naming consists in a comparison of the Berinmo and Yêlì Dnye hue naming centroids with the MacLaury red, yellow, green and blue elemental hues.

In the Yêlì Dnye study, Levinson elicited names for the full Lenneberg and Roberts (1956) array of forty equally spaced Munsell hues at eight levels of lightness (Munsell value), plus achromatic chips at ten levels of lightness.[7] (The full array of 330 colors is shown schematically in Figure 1). In the Berinmo study, RDD elicited names for a 20-hue array at eight levels of lightness which Rosch had used with the Dani. (The resulting 160 cell hue/lightness array omits every other hue column of

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[6]Interpreting the size of interchip intervals in Munsell space as reflecting psychological distance is not generally accepted (see, *e.g.*, Boynton 1997: 139). Nevertheless, since RDD make this kind of interpretation throughout their paper, I have followed their practice in the interests of consistency, and have calculated the centroids of Berinmo and Yêlì Dnye naming responses on the maximal-saturation surface of the Munsell solid in the hue and value dimensions. It has not been possible to include centroid calculations for achromatic colors (specifically, black and white) for technical reasons involving the particular sets of stimuli selected from the full Munsell set. Although centroid calculations for naming responses employing the black and white terms of Bernimo and Yêlì Dnye cannot be made from the available data, it seems clear that if statistically usable naming data for these terms were available, they would – by virtue of the fact that they patently *are* black and white terms – strengthen the case for universal constraints on color naming.

[7]This same array was used by Berlin and Kay (1969), the World Color Survey (Kay *et al.* 1997), and The Mesoamerican Color Survey (MacLaury 1997).

the 320 cell hue/lightness array.)[8] In depicting the Berinmo hue naming centroids below, I have transformed the Berinmo 160 cell hue/lightness space into the 320 cell hue/lightness space, interpolating as necessary.

FIGURE 1 HERE

RDD do not report having used achromatic stimuli in their naming task, but they write: “Berinmo has five basic color terms ... The names of the Berinmo terms are *wapa* (both the term for a European person and for white and all very pale colors); *kel* (the term for black, for charcoal or anything burnt, but also meaning dirty); *mehi* (the term for red and for the color of the fruit of the red Pandanus palm); *wor* (the term for leaves ready to fall from a tree and covering a range of yellow/orange/brown and khaki) and *nol* (the term meaning live and covering green/yellow-green/blue/and purple” (pp. 371-2). These glosses indicate that (*nol*), the term including green, blue and purple, extends further into yellow than does English green: *nol* is reported to include "yellow-green." *wor* – which includes focal yellow – is not said to include any shades including a green component. The naming data tabulated in RDD's Figure 2 (p.373), present a contradictory picture as regards the *wor/nol* boundary. In Figure 2 the yellow term, *wor*, intrudes into the area that would be called green in familiar languages, and the extension of the *nol* term is concomitantly retracted to exclude yellowish greens. For example, if one compares RDD's Figure 2 for Berinmo naming with their Figure 1, which reproduces Rosch's English naming results, one finds that every English green chip bordering yellow in English (Figure 1) is included in Berinmo *wor* ('yellow') in

[8]Both Levinson and RDD presented chips for naming one by one. Levinson presented chips in the constant random order employed in the WCS. RDD randomized chip order for each subject. Both checked subjects for color vision deficiencies using the Ishihara plates and found none. Levinson tested seven Yêlì Dnye subjects, RDD twenty-two Berinmo subjects. Levinson used natural lighting. RDD employed a specially constructed display box which produced a light equivalent to CIE illuminant C.

Figure 2 and similarly no English yellow chip is included in Berinmo *nol* ('green or blue or purple'). A number of RDD's experiments involve the relative placement of the English yellow / green and Berinmo *wor/nol* boundaries. In the remainder of this paper I ignore RDD's glosses with regard to the *wor/nol* boundary, basing all calculations on their tabulations of actual naming responses.

Notwithstanding difficulties regarding the *wor/nol* boundary, RDD's glosses suggest unequivocally that Berinmo subjects would reliably name white and black chips *wapa* and *kel*, respectively, were such chips presented to them. (Levinson, as well, reports Yêlì Dnye basic color terms for black and white.) Berinmo *mehi* appears to be an unremarkable term for red. The Berinmo *wor* term, focused in yellow and extended into orange, brown and khaki, has many analogues among World Color Survey (WCS)[9] and Berlin and Kay languages. Judging from the RDD's Figure 2 – not their glosses – the extension of the term including focal yellow into the yellowish green area is similar to the situation in Hanuno'o where the a red / yellow / orange / brown term, (ma)rara', also extends into the yellowish green area. The remaining grue term, *nol*, focused in green, is extended through purple, a common occurrence in the WCS data (Kay and Berlin 1997: 200).

On the basis of inspection of the original Berlin and Kay data, the Mesoamerican Color Survey data and the WCS data, MacLaury (1997: 467) has proposed four chips in the 320 Munsell hue / lightness space as representing the universal elemental hues. These are shown in Figure 2. Naming centroids have

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[9] The World Color Survey, directed by Brent Berlin, Paul Kay and William Merrifield, collected – with the much appreciated cooperation of many missionary linguists of the Summer Institute of Linguistics (now SIL International) – color naming data from 110 languages around the world, averaging 24 speakers per language, insofar as possible monolingual, *in situ,* using the native language as the language of interview. 330 color chips were presented in a fixed random order for naming by the local speaker and best example ("focal") choices were elicited from a full array of the stimuli following the naming task. The focal responses do not figure in this report, only the centroids of the naming responses.

been calculated for each of the chromatic terms for Yêlì Dnye from the data presented in Levinson (2000, Figs. 2, 3, and 4) and for Berinmo from unpublished data generously furnished by Roberson, Davies and Davidoff and are shown on the following figures.

FIGURE 2 HERE

Figure 3 shows the location of the naming centroids for Yêlì Dnye red, green and yellow expressions elicited by Levinson, along with the MacLaury elemental hues. Levinson mentions a class of blue-denoting expressions, which he characterizes as the least lexicalized, least reliable, and least generally shared of the hue expression classes elicited in the naming task. Although the focal choices for the blue expressions are reported (Levinson 2000, Figure 5) the naming responses are not. Hence no centroid calculation for the Yêlì Dnye blue naming data is possible. Impressionistically, prediction of the location of the Yêlì Dnye hue centroids from the MacLaury elemental chromatic colors and the UE model appears confirmed.

FIGURE 3 HERE

Berinmo hue naming centroids were also calculated and compared with the MacLaury elementals and the Yêlì Dnye naming centroids. In deriving a prediction for the naming centroid of the *nol* term in Berinmo, account must be taken of the fact that this is a grue term, not a green term. Consequently we predict the centroid of the naming responses for this term to fall half way between elemental green and elemental blue. Figure 4 compares the Berinmo hue naming centroids with the MacLaury elemental chromatic colors and the Yêlì Dnye hue naming centroids. We see that the naming centroid for *nol* is, as predicted, the chip half way between the green and blue elementals. Again, prediction of local hue naming responses from universally posited elemental colors is impressionistically confirmed. **In Figure 4 each of the six naming centroids of the two languages is observed to be a chip either**

identical to or adjacent to a chip predicted by the UE model using the MacLaury elemental chromatic colors.

FIGURE 4 HERE

Table 1 gives the comparison of the elemental hues and the hue naming centroids of the two languages in tabular form.

TABLE 1 HERE

3. Statistical assessment of impressionistic findings

The impression that the Berinmo and Yêlì Dnye hue naming data are closer to universal tendencies than would be expected by chance is now subjected to statistical test. Since Berinmo and Yêlì Dnye have been claimed to provide challenges to the UE model, no unfair advantage for the UE hypothesis is gained by assuming these languages to be representative of the world's languages. Both languages yield three hue centroids. We are thus looking for the joint probability of two independent events; each of these events involves for a particular language with three hue naming centroids both the centroid pattern observed for that language, π_o, and the centroid pattern predicted for that language by the UE model and the Maclaury elementals π_e. Specifically, for each language separately we need to calculate the probability p that, if each of the 320 cells has an equal probability of receiving a hue centroid, a pattern of three hue centroids will be observed that is as close to π_e as π_o is. The probability of the joint, two-language event will then be the product of the values of p for the two languages. (It turns out that p values for the two languages are the same, so we will only have to make one calculation of p and square that result.)

FIGURE 5 HERE

The calculation of p and p^2 is shown in Figure 5. We first quantify the idea of how close pattern π_o is to pattern π_e. To do this we imagine the 320-cell hue chart curled into a cylindrical grid with 40 columns and 8 rows, where the leftmost and rightmost columns of the rectangular chip array are adjacent columns on the cylinder. Berinmo has three basic hue terms. Yêlì Dnye has either three or four established hue categories; in either case, we have naming data for three: red, yellow and green. In the case of Yêlì Dnye the null hypothesis is that the hue naming centroids will bear no particular relation to the red, yellow and green elementals and the alternative, UE, hypothesis is that the former will be “close” to the latter. As an initial quantification of “closeness” between π_e and π_o, we designate a target area for each predicted chip to consist of the 3x3 chip minimal square that consists of the predicted chip and the eight chips surrounding it. Assuming the UE predicted *nol* ‘grue’ target for Berinmo to be the chip exactly intermediate between universal green and universal blue (F23, 7.5BG/5), the UE prediction that every naming centroid falls in a distinct target area is exactly confirmed for each language, as we noted in connection with Figure 4 and Table 2. What is the probability that three cells chosen at random from the 320 cell array will fall, one each, in the three target areas? That probability p is given by

(1) $p = 27/320 \times 18/320 \times 9/320 = 4{,}374/32{,}768{,}000 = .00013$

The probability p represents the chance occurrence of three observed centroids in a single language landing in each of three distinct nine-cell, non-overlapping target areas, when each of the 320 cells of the space has an equal probability of being hit. For two independently chosen, three-hue-term languages, the probability of both achieving success by this criterion is

(2) $p^2 = .00000002.$

In general, for n cells and t target areas, each of size s, the probability P of t successes in choosing t cells is given by

(3) $$P = \prod_{i=1}^{t} s_i/n$$

The most stringent, least powerful, test – other than one requiring direct hits on single predicted chips – is the one we have just assessed, where the size s of the target square is nine. (See Table 2) The next largest target square has five cells to a side, s = 25, and the next largest after that has seven cells to a side, s = 49. (Nine cells to a side, s = 81, would cause overlapping target areas.) Whichever test we choose, the result in favor of the UE hypothesis of closeness of observed hue centroids to predictions from UE theory is significant, as shown in Table 2. Statistical testing thus confirms the impression of Figure 4 and Table 2, that the observed locations of naming centroids for Berinmo and Yêlì Dnye hue terms are closer to those predicted by the UE model and the proposed universal elemental chromatic colors than would be expected by chance.

TABLE 2 HERE

4. Conclusion

Berinmo and Yêlì Dnye have been claimed elsewhere to present special problems for the UE theory of color naming universals. The present analysis of the color naming patterns in these two languages fails to support such arguments and rather supports the UE hypothesis of universal constraints on cross language color naming.[10] RDD showed that the performance of Berinmo subjects in a variety of experimental tasks involving colors and color words failed to replicate the performance of Rosch's Dani subjects. They also show that where the boundaries between color terms differ in Berinmo and English these differences correlate with differences in color cognition: Berinmo speakers exhibit enhanced color

[10] A variety of hypotheses have been advanced to explain these cross-linguistic constraints on color naming, several combining one or another general principle of economy of categorization with some mechanism that picks out special colors e.g., whatever mechanism produces the Hering opponent fundamentals (Kay and Maffi 1997), the differential frequencies with which colors occur in natural scenes (Yendrikohovskij 2001), differential color saliencies arising from irregularities in the perceptual color solid (Jameson and D'Andrade 1997).

discrimination from memory across Berinmo category boundaries – but not across English boundaries – while English speakers show the reverse pattern. Kay and Kempton (1984) obtained an analogous result for simultaneous judgments of similarity among color chips. These judgments were shown to be influenced by language-specific lexical boundaries in a comparison of English speakers with speakers of Tarahumara, an unaffiliated language of Mexico that lacks a lexical distinction between green and blue. Demonstrations such as these, which show that differences in color naming between languages can influence non-linguistic behavior toward colors, should not, however, be taken as evidence that color naming varies without constraint across the world's languages. Berlin and Kay (1969) emphasized the fact that different languages have different numbers of basic color terms and so color term boundaries cannot be universal. Berinmo color naming does not depart in any major respect from that found in other languages of the world with five major color terms. RDD's results appear to be consistent with Davies's view that "the inferential load [that Rosch's Dani work] is required to support is too heavy" (Davies 1997: 186). Neither the Berinmo nor the Yélî Dnye data, however, weaken the hypothesis that there exist universal constraints on cross-language color naming; indeed, they strengthen it.

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Figures and Tables

VALUE (LIGHTNESS) →

HUE ↓		2	3	4	5	6	7	8	9	
H	2.5R									1
U	5R									2
E	7.5R									3
	10R									4
↓	2.5YR									5
	5YR									6
	7.5YR									7
	10YR									8
	2.5Y									9
	5Y									10
	7.5Y									11
	10Y									12
	2.5GY									13
	5GY									14
	7.5GY									15
	10GY									16
	2.5G									17
	5G									18
	7.5G									19
	10G									20
	2.5BG									21
	5BG									22
	7.5BG									23
	10BG									24
	2.5B									25
	5B									26
	7.5B									27
	10B									28
	2.5BP									29
	5BP									30
	7.5PB									31
	10BP									32
	2.5P									33
	5P									34
	7.5P									35
	10P									36
	2.5RP									37
	5RP									38
	7.5RP									39
	10RP									40
		I	H	G	F	E	D	C	B	

Figure 1. Schematic Array of 320 Munsell Chromatic Colors. Munsell notations at top and left; WCS notations at bottom and right. All chips at maximum chroma (saturation). [A pretty good reproduction of this stimulus array can be viewed online at http://www.ICSI.Berkeley.EDU/wcs/study.html

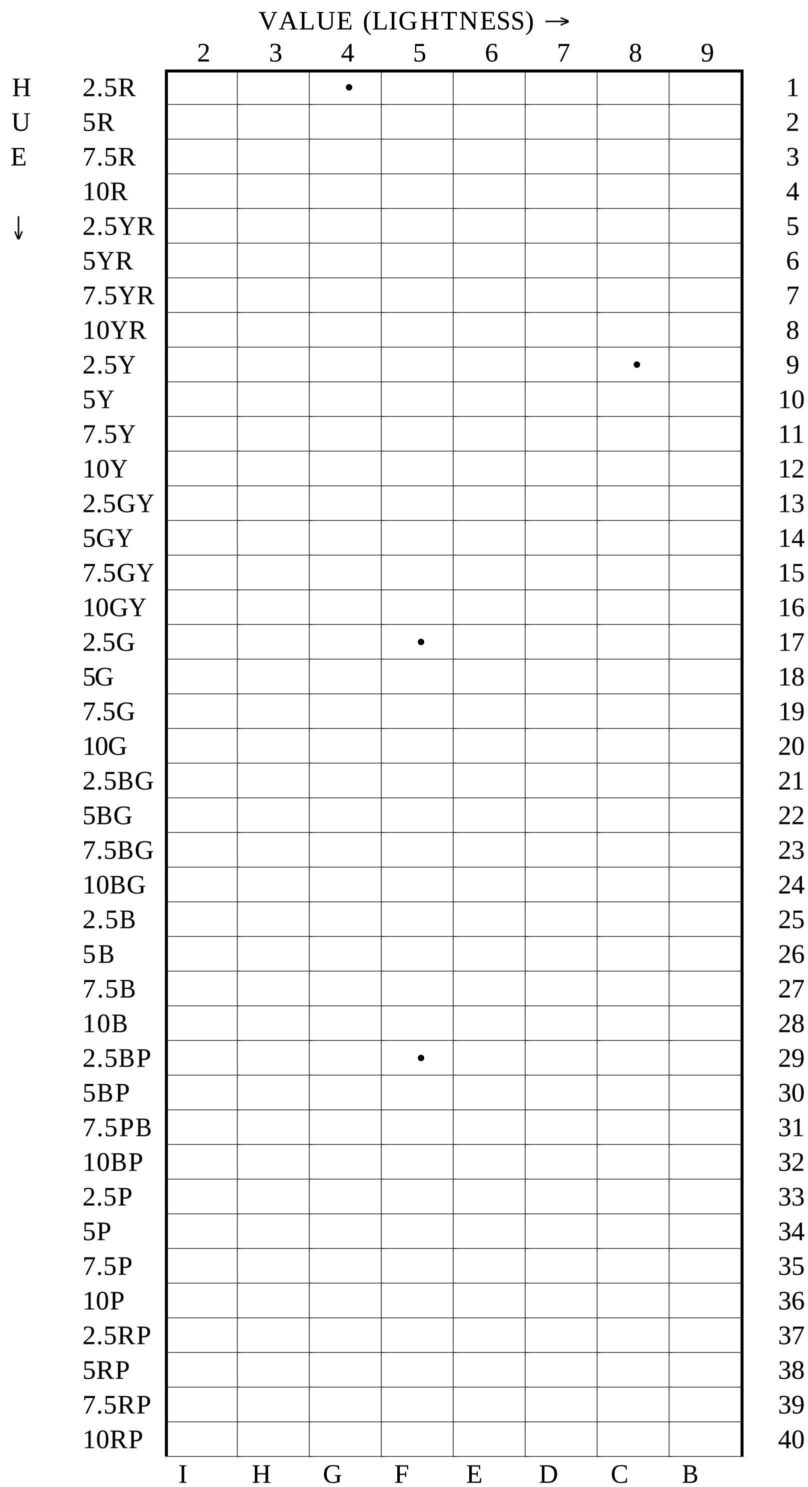

Figure 2. Elemental Chromatic Colors (indicated by •)[11]. (See Figure 1 for detailed legend.)

[11] Source: MacLaury (1997: 467)

VALUE (LIGHTNESS) →

HUE ↓	2	3	4	5	6	7	8	9	
2.5R			•Y						1
5R									2
7.5R									3
10R									4
2.5YR									5
5YR									6
7.5YR									7
10YR									8
2.5Y							•		9
5Y						Y			10
7.5Y									11
10Y									12
2.5GY									13
5GY									14
7.5GY									15
10GY									16
2.5G				•					17
5G				Y					18
7.5G									19
10G									20
2.5BG									21
5BG									22
7.5BG									23
10BG									24
2.5B									25
5B									26
7.5B									27
10B									28
2.5BP				•					29
5BP									30
7.5PB									31
10BP									32
2.5P									33
5P									34
7.5P									35
10P									36
2.5RP									37
5RP									38
7.5RP									39
10RP									40
	I	H	G	F	E	D	C	B	

Figure 3. Elemental Chromatic Colors (indicated by •) and Yêlì Dnye Naming Centroids (indicated by Y). (See Figure 1 for detailed legend.)

VALUE (LIGHTNESS) →

HUE ↓	2	3	4	5	6	7	8	9	
2.5R			•Y	B					1
5R									2
7.5R									3
10R									4
2.5YR									5
5YR									6
7.5YR									7
10YR									8
2.5Y							•		9
5Y						YB			10
7.5Y									11
10Y									12
2.5GY									13
5GY									14
7.5GY									15
10GY									16
2.5G				•					17
5G				Y					18
7.5G									19
10G									20
2.5BG									21
5BG									22
7.5BG				*B					23
10BG									24
2.5B									25
5B									26
7.5B									27
10B									28
2.5BP				•					29
5BP									30
7.5PB									31
10BP									32
2.5P									33
5P									34
7.5P									35
10P									36
2.5RP									37
5RP									38
7.5RP									39
10RP									40
	I	H	G	F	E	D	C	B	

Figure 4. Elemental Chromatic Colors (indicated by •), Yêlì Dnye Naming Centroids (indicated by Y), Predicted grue centroid (indicated by *), and Berinmo Naming Centroids (indicated by B). (See Figure 1 for detailed legend.)

Elemental Hues	Red G1 (2.5R/4)	Yellow C9 (2.5Y/8)	Green F17 (2.5G/5)	Blue F29 (2.5PB/5)
Berinmo	*mehi* 'red' F1 (2.5R/5)	*wor* 'yellow' D10 (5Y/7)	*nol* 'grue' F23 (7.5BG/5)	
Yélî Dnye	red terms G1 (2.5R/4)	yellow terms D10 (5Y/7)	green terms F18 (2.5G/4)	—

Table 1: Berinmo and Yélî Dnye Hue Naming Centroids Compared to MacLaury's Universal Elemental Hues

Given:

(1) a cylindrical surface marked off into 40 columns and 8 rows, producing 320 cells,

(2) designation of 3, non-overlapping target regions, each containing 9 cells;

Find:

the probability p that each of 3 cells chosen at random will belong to a distinct target region.

$p = 27/320 \times 18/320 \times 9/320 = 4{,}374 \ / \ 32{,}768{,}000 = .00013$

$p^2 = .00000002$

In general, for n total cells and t target areas, each of size s, the probability p of t successes when choosing t cells is

$$\prod_{i=1}^{t} si/n$$

Figure 5. Framing the problem in probability terms

For s = 9: p = .00013; p^2 = .00000002.
For s = 25: p = .00286; p^2 = .000008.
For s = 49: p = .02154; p^2 = .0005.

(s ≥ 81 would cause overlap in target areas.)

Table 2: Varying the size of the target region