Differential Evolution Design of an IIR-Filter with Requirements for Magnitude and Group Delay

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Abstract

The task of designing an 18 parameter IIR-filter which has to meet tight specifications for both magnitude response and group delay is investigated. This problem must usually be tackeled by specialized design methods and requires an expert in digital signal processing for its solution. The usage of the general purpose minimization method Differential Evolution (DE), however, allows to perform the filter design with a minimum knowledge about digital filters.

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1. Introduction

IIR filters are generally applied in cases where tight requirements for the magnitude response are imposed upon the filter while phase or group delay don't play a major role. Appropriate design algorithms are widely available for this problem domain [1], [2].

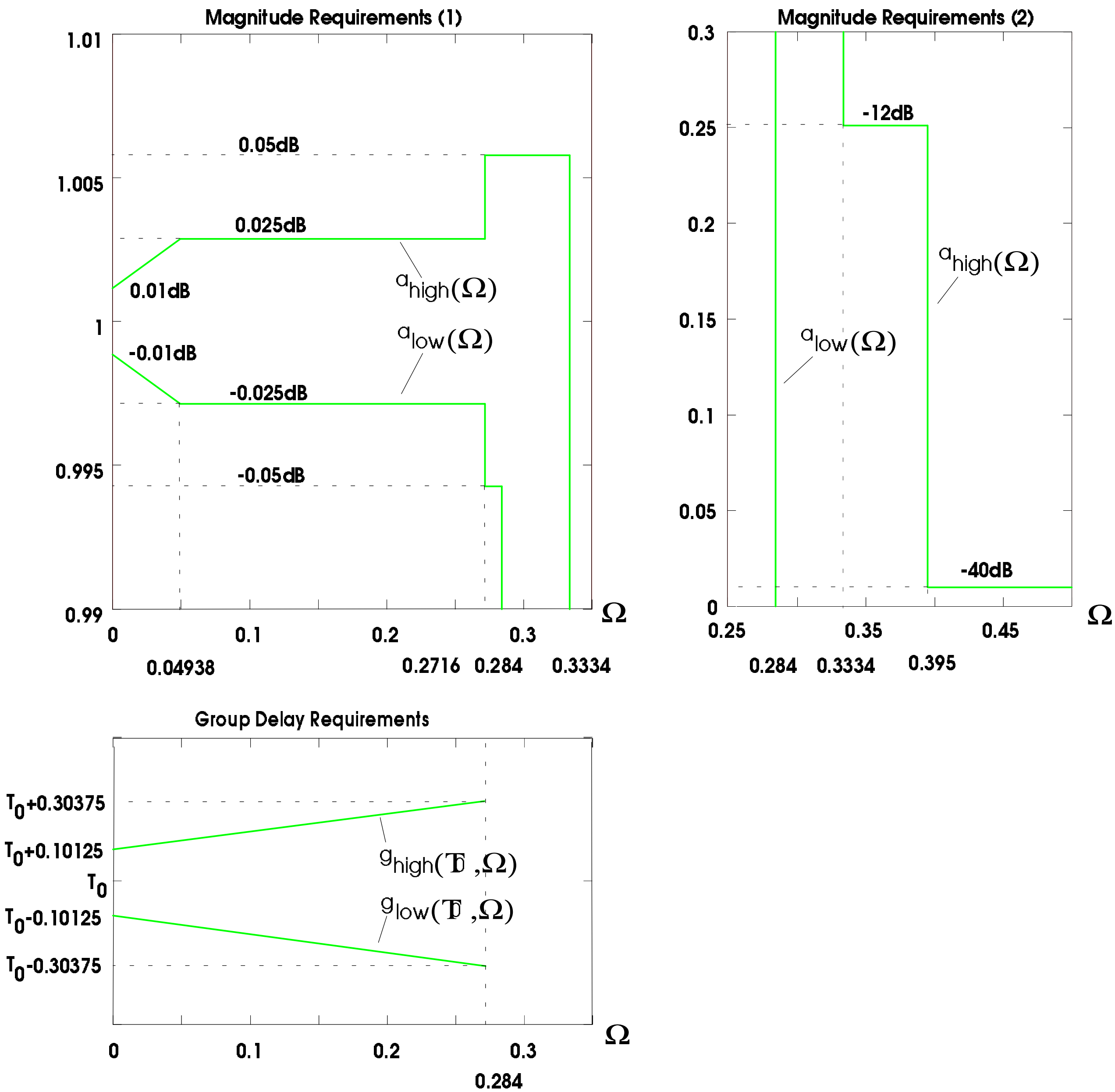

Fig. 1: Tolerance schemes for magnitude and group delay of a graphics codec [3].

In cases where linear or quasi linear phase is required one usually resorts to FIR filters which can provide piecewise exactly linear phase, i.e. piecewise constant group delay. Linearity of phase, however, is only required in the pass band of a filter which is why for some applications the usage of FIR filters means an unnecessary high realization expense. An example for this is a graphics codec for sample rate reduction [3], the magnitude and group delay specifications of which are depicted in Fig. 1.

The specifications of Fig. 1 can be fulfilled by employing a linear phase FIR filter of degree 61 [4]. As the tight passband specifications for the magnitude can be fulfilled with an IIR filter of comparably low degree, it seems worthwhile trying to meet the specifications of Fig. 1 with an IIR approach even though the group delay specifications will enforce a filter degree which is higher than the one needed for just fulfilling the magnitude requirements. The hope is to achieve an overall reduced realization expense and a lower overall filter delay. In fact a successful design of this kind has been reported in [4] where the graphics codec had been realized by a linear phase FIR prefilter of degree 7 followed by an IIR filter of degree 14. The IIR filter consists of an IIR lowpass of degree 6 which fulfills the magnitude requirements and an IIR allpass of degree 8 which is used to make the entire filter meet the group delay specifications. It has been shown in [5] that via specialized design methods for IIR filters which simultaneously consider magnitude and phase requirements the IIR filter degree can be reduced to 10.
Specialized methods for IIR filter design in the complex domain, however, are usually not readily available and have the disadvantage of being difficult to implement. In an industrial development process this can be a major obstacle if these specialized techniques and the necessary expertise are not available. The development of an appropriate tool requires considerable effort and can quickly turn out to be a project of its own, requiring a substantial amount of development time and cost. Hence it is attractive to perform the filter design with a general purpose function minimizer which is powerful enough to solve truly difficult design problems and yet is simple to understand and to implement. Differential Evolution (DE) [6] has recently proven to be fast converging for a wide variety of test cases while its implementation takes less than 300 lines of C source code. In the following chapters the steps to synthesize the graphics codec by using DE are elaborated. Chapter 2 provides the necessary filter equations which are used to transform the filter design problem into a objective function minimization problem. This transformation is described in chapter 3. Chapter 4 introduces the method of DE while Chapter 5 eventually presents the design results obtained.

2. IIR Filter Equations

The analysis of IIR filters via the z-Transform is well treated in the signal processing literature [7] which is why only the necessary equations will be given in the following. The transfer function H(z) of an IIR filter

is defined by
$$H(z)=\frac{\sum_{n=0}^{N}a_n\cdot z^{-n}}{1+\sum_{m=1}^{M}b_m\cdot z^{-m}}=a_0\cdot\frac{\prod_{n=1}^{N}(z-z_{0_n})}{\prod_{m=1}^{M}(z-z_{p_m})}\cdot z^{M-N} \tag{1}$$
with
$$z=e^{j2\pi\Omega} \tag{2}$$
and
$$\Omega=\frac{\omega}{\omega_A}. \tag{3}$$
ω is called the radian frequency and ω_A the radian sampling frequency. The transfer function (1) exhibits the zeroes
$$z_{0_n}=r_{0_n}\cdot e^{j2\pi\Phi_{0_n}},\ n=1,2,\ldots,N \tag{4}$$
as well as the poles
$$z_{p_m}=r_{p_m}\cdot e^{j2\pi\Phi_{p_m}},\ m=1,2,\ldots,M. \tag{5}$$
The magnitude response $A(\Omega)$ of the IIR filter can be computed according to
$$A(\Omega)=\left|H(e^{j2\pi\Omega})\right|=a_0\cdot\frac{\prod_{n=1}^{N}\sqrt{1-2r_{0_n}\cdot\cos(2\pi(\Omega-\Phi_{0_n}))+r_{0_n}^2}}{\prod_{m=1}^{M}\sqrt{1-2r_{p_m}\cdot\cos(2\pi(\Omega-\Phi_{p_m}))+r_{p_m}^2}} \tag{6}$$
and the group delay $G(\Omega)$ via
$$\begin{aligned}G(\Omega)=-\frac{1}{2\pi}\cdot\frac{d}{d\Omega}\left(\mathrm{arc}H(e^{j2\pi\Omega})\right)&=\sum_{m=1}^{M}\frac{1-r_{p_m}\cdot\cos(2\pi(\Omega-\Phi_{p_m}))}{1-2r_{p_m}\cdot\cos(2\pi(\Omega-\Phi_{p_m}))+r_{p_m}^2}\\&-\sum_{n=1}^{N}\frac{1-r_{0_n}\cdot\cos(2\pi(\Omega-\Phi_{0_n}))}{1-2r_{0_n}\cdot\cos(2\pi(\Omega-\Phi_{0_n}))+r_{0_n}^2}+(M-N)\end{aligned}. \tag{7}$$
For the case that (1) exhibits only real filter coefficients a_n, b_m, the zeroes and poles of $H(z)$ are either real or appear in conjugate complex pairs. It is common practice to decompose an IIR filter into a cascade of subfilters of first and second degree, rendering
$$H(z)=a_0\prod_{k=1}^{K}H_k(z). \tag{8}$$
If only real coefficients are assumed and $H_k(z)$ is a filter of first degree, we obtain
$$A_{k,1}(\Omega)=\left|H_k(e^{j2\pi\Omega})\right|=\frac{\sqrt{1-2r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))+r_{0_k}^2}}{\sqrt{1-2r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))+r_{p_k}^2}} \tag{9}$$
and
$$\begin{aligned}G_{k,1}(\Omega)=-\frac{1}{2\pi}\cdot\frac{d}{d\Omega}\left(\mathrm{arc}H_k(e^{j2\pi\Omega})\right)&=\frac{1-r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))}{1-2r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))+r_{p_k}^2}\\&-\frac{1-r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))}{1-2r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))+r_{0_k}^2}\end{aligned}. \tag{10}$$
If $H_k(z)$ is a filter of second degree, magnitude and group delay are computed according to

$$A_{k,2}(\Omega) = \frac{\sqrt{1-2r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))+r_{0_k}^2}}{\sqrt{1-2r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))+r_{p_k}^2}} \cdot \frac{\sqrt{1-2r_{0_k}\cdot\cos(2\pi(\Omega+\Phi_{0_k}))+r_{0_k}^2}}{\sqrt{1-2r_{p_k}\cdot\cos(2\pi(\Omega+\Phi_{p_k}))+r_{p_k}^2}} \qquad (11)$$

and

$$\begin{aligned} G_{k,2}(\Omega) = & \frac{1-r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))}{1-2r_{p_k}\cdot\cos(2\pi(\Omega-\Phi_{p_k}))+r_{p_k}^2} + \frac{1-r_{p_k}\cdot\cos(2\pi(\Omega+\Phi_{p_k}))}{1-2r_{p_k}\cdot\cos(2\pi(\Omega+\Phi_{p_k}))+r_{p_k}^2} \\ & - \frac{1-r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))}{1-2r_{0_k}\cdot\cos(2\pi(\Omega-\Phi_{0_k}))+r_{0_k}^2} - \frac{1-r_{0_k}\cdot\cos(2\pi(\Omega+\Phi_{0_k}))}{1-2r_{0_k}\cdot\cos(2\pi(\Omega+\Phi_{0_k}))+r_{0_k}^2} \end{aligned} . \qquad (12)$$

3. Design of the Objective Function

Like in [5] the graphics codec specifications from Fig. 1 shall be met by a cascade of a seventh order linear phase FIR pre-filter and an IIR filter of order 10 with an overall magnitude response

$$A_{GC}(\Omega) = \sum_{\nu=0}^{3} f_\nu \cdot 2\cos(\pi\Omega(\nu-3.5)) \cdot \frac{\prod_{i=1}^{5}\sqrt{1-2r_{0_i}\cdot\cos(2\pi(\Omega-\Phi_{0_i}))+r_{0_i}^2}\cdot\sqrt{1-2r_{0_i}\cdot\cos(2\pi(\Omega+\Phi_{0_i}))+r_{0_i}^2}}{\prod_{j=1}^{4}\sqrt{1-2r_{p_j}\cdot\cos(2\pi(\Omega-\Phi_{p_j}))+r_{p_j}^2}\cdot\sqrt{1-2r_{p_j}\cdot\cos(2\pi(\Omega+\Phi_{p_j}))+r_{p_j}^2}} \cdot a_0$$

(13)

and the group delay

$$\begin{aligned} G_{GC}(\Omega) = 1.75 & - \sum_{i=1}^{5} \frac{1-r_{0_i}\cdot\cos(2\pi(\Omega-\Phi_{0_i}))}{1-2r_{0_i}\cdot\cos(2\pi(\Omega-\Phi_{0_i}))+r_{0_i}^2} + \frac{1-r_{0_i}\cdot\cos(2\pi(\Omega+\Phi_{0_i}))}{1-2r_{0_i}\cdot\cos(2\pi(\Omega+\Phi_{0_i}))+r_{0_i}^2} \\ & + \sum_{j=1}^{4} \frac{1-r_{p_j}\cdot\cos(2\pi(\Omega-\Phi_{p_j}))}{1-2r_{p_j}\cdot\cos(2\pi(\Omega-\Phi_{p_j}))+r_{p_j}^2} + \frac{1-r_{p_j}\cdot\cos(2\pi(\Omega+\Phi_{p_j}))}{1-2r_{p_j}\cdot\cos(2\pi(\Omega+\Phi_{p_j}))+r_{p_j}^2} \end{aligned} . \qquad (14)$$

Like in [5], the FIR filter coefficients remain fixed to the values

$$f_\nu = \{-0.033271, -0.019816, 0.169865, 0.415454\} \qquad (15)$$

leaving only the radii and angles of poles and zeroes of the IIR part as well as the constant a_0 as free parameters. However, the value a_0 in (13) is a redundant parameter and should be fixed during the optimization process. Otherwise the optimization procedure has infinitely many global optima to choose from. Hence the only parameters that are varied in the objective function are the radii and the angles of zeroes as well as poles of the IIR filter. Note that the time constant T_0 in Fig. 1 is not required to have a specific value and therefore will be part of the result of the design process rather than a condition to be fulfilled.

As the IIR filter shall be designed by employing a general purpose function minimizer, the design problem has to be restated as an objective function the minimization of which yields the desired solution. The main part of the objective function has been chosen to be very simple. Basically it takes on the maximum absolute deviation of either the magnitude response (13) or the group delay (14) from the corresponding specifications in Fig. 1, whichever deviation is greater. In addition several penalties are included in the objective function which reflect special knowledge of the filter design problem. These penalties realize the following constraints:

1) All parameters have to be positive. Although this restriction is not necessary for the angles Φ_{0_i}, Φ_{p_j}, it avoids redundant angle values.
2) The radii r_{p_j} of the poles must be < 1 to ensure stability of the IIR filter.
3) The angles Φ_{p_j} of the poles must be ≤ 0.284 as it doesn't make sense to locate poles in the transition band or the stop band of a filter.
4) The radii r_{0_i} of the zeroes must be ≥ 1 to allow for phase compensation in the passband.
5) The angles Φ_{0_i} of the zeroes must be ≤ 0.5 in order to avoid redundant angle values.

Each vilolation of a penalty will be considered by adding a positive value to the objective function. The value itself depends on the extent of the violation which is penalized. The greater the violation the larger the penalty value will be chosen. We now can build the objective function f(x) where x is the parameter vector

$$\underline{x} = (x_1, x_2, \ldots, x_{18})^T \tag{16}$$

with the mapping

$$x_i = r_{0_i}, \quad i = 1,2,3,4,5 \tag{17}$$

$$x_{i+5} = \Phi_{0_i}, \quad i = 1,2,3,4,5 \tag{18}$$

$$x_{i+10} = r_{p_i}, \quad i = 1,2,3,4 \tag{19}$$

and

$$x_{i+14} = \Phi_{p_i}, \quad i = 1,2,3,4 \tag{20}$$

The mathematical formulation of the objective function f(x) is

$$f(\underline{x}) = \max\left(\mathrm{dev}(A_{GC}(\Omega)), \mathrm{dev}(G_{GC}(\Omega))\right) + \sum_{\mu=1}^{5} P_\mu \tag{21}$$

where

$$\mathrm{dev}(A_{GC}(\Omega)) = \max\begin{cases} A_{GC}(\Omega) - a_{high}(\Omega) & \text{if} \quad A_{GC}(\Omega) > a_{high}(\Omega), \quad 0 \quad \text{else} \\ a_{low}(\Omega) - A_{GC}(\Omega) & \text{if} \quad A_{GC}(\Omega) < a_{low}(\Omega), \quad 0 \quad \text{else} \end{cases} \tag{22}$$

and

$$\mathrm{dev}(G_{GC}(\Omega)) = \max\begin{cases} G_{GC}(\Omega) - g_{high}(T_0,\Omega) & \text{if} \quad G_{GC}(\Omega) > g_{high}(T_0,\Omega), \quad 0 \quad \text{else} \\ g_{low}(T_0,\Omega) - G_{GC}(\Omega) & \text{if} \quad G_{GC}(\Omega) < g_{low}(T_0,\Omega), \quad 0 \quad \text{else} \end{cases}. \qquad (23)$$

In (23) T_0 is always computed such that the maximum deviation from the requirements in Fig. 1 is minimal. The penalty terms P_μ in (21) are computed via

$$P_1 = \sum_{i=1}^{18} \begin{cases} 20000 - 100 \cdot x_i & \text{if} \quad x_i < 0 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(all parameters > 0)} \qquad (24)$$

$$P_2 = \sum_{i=1}^{5} \begin{cases} 2 + \dfrac{100}{x_i + 10^{-10}} & \text{if} \quad x_i < 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(radii of zeroes} \geq 1\text{)} \qquad (25)$$

$$P_3 = \sum_{i=6}^{10} \begin{cases} 2 + 100 \cdot x_i & \text{if} \quad x_i > 0.5 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(angles of zeroes} \leq 0.5\text{)} \qquad (26)$$

$$P_4 = \sum_{i=11}^{14} \begin{cases} 2 + 100 \cdot x_i & \text{if} \quad x_i \geq 1. \\ 0 & \text{otherwise} \end{cases} \qquad \text{(radii of poles < 1)} \qquad (27)$$

$$P_5 = \sum_{i=15}^{18} \begin{cases} 2 + 100 \cdot x_i & \text{if} \quad x_i > 0.284 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(angles of poles} \leq 0.284\text{)} \qquad (28)$$

and correspond to the verbal penalty descriptions given above.

A parameter vector $\underline{x}$ for which the objective function $f(\underline{x})$ is zero solves the filter design problem.

4. Differential Evolution for Function Minimization

The objective function (21) is a highly nonlinear and partly non-differentiable function which exhibits many local minima. Therefore a powerful global minimization algorithm is required which is able to cope with these properties. Recently Differential Evolution (DE) [6] has proven to be a promising candidate for solving such problems. As an additional benefit DE is very simple to understand and to implement. DE is also particularly easy to work with as only few control variables exist which remain fixed throughout the entire optimization procedure. The method described in the following is one of several variants of DE which, however, differ only slightly.

DE is a parallel direct search method which utilizes NP D-dimensional parameter vectors

$$\underline{x}_{i,G}, \ i = 1, 2, 3, \ldots, NP. \qquad (29)$$

as a population for each generation G, i.e. for each iteration of the optimization process. NP doesn't change during the minimization process. The initial population is chosen randomly if no assumptions about the design solution are made. As a rule, we will assume a uniform probability distribution for all random decisions unless otherwise stated. In case a preliminary design solution is available, the initial population is often generated by adding normally distributed random deviations to the nominal solution $\underline{x}_{nom,1}$. The crucial idea behind DE is a new scheme for generating trial parameter vectors. DE generates new parameter vectors basically by adding the weighted difference vector between two population members to a third member. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector replaces the vector with which it was compared. In addition the best parameter vector $\underline{x}_{best,G}$ is evaluated for every generation G in order to keep track of the progress that is made during the minimization process. The special version of DE which was used to design the graphics codec is described below.

For each vector $\underline{x}_{i,G}$, i = 1,2,3,...,NP, a trial vector $\underline{v}_{i,G+1}$ is generated according to

$$\underline{v}_{i,G+1} = \underline{x}_{i,G} + F \cdot (\underline{x}_{best,G} - \underline{x}_{i,G}) + F \cdot (\underline{x}_{r_1,G} - \underline{x}_{r_2,G}) \quad , \tag{30}$$

with $r_2, r_3 \in [1, NP]$, integer and mutually different, and F > 0.

(13)

The integers r_1, r_2 are chosen randomly from the interval [1, NP] and are different from the running index i. F is a real and constant factor $\in$ [0, 1] which controls the inclusion of the current best vector $\underline{x}_{best,G}$ as well as the amplification of the differential variation $(\underline{x}_{r_1,G} - \underline{x}_{r_2,G})$. The search procedure centers the more around the current best vector the closer F is set to 1. At the same time the weight of the differential variation $(\underline{x}_{r_1,G} - \underline{x}_{r_2,G})$ is increased in order prevent the search from getting too local and greedy. Fig. 2 shows a two dimensional example that illustrates the different vectors which play a part in the vector generation scheme.

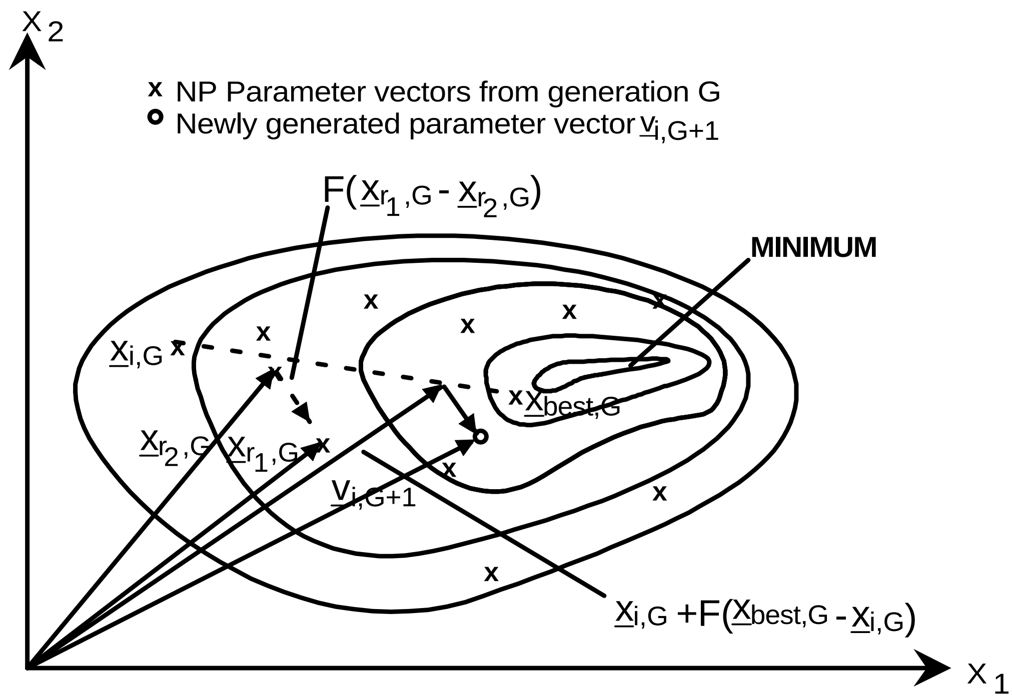

Fig.2: Two dimensional example of an objective function showing its contour lines and the process for generating $\underline{v}_{i,G+1}$ for DE.

In order to increase the diversity of the new parameter vectors, the vector

$$\underline{u}_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \ldots, u_{Di,G+1})^T \tag{31}$$

with

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{for} \quad j = \langle n \rangle_D, \langle n+1 \rangle_D, \ldots, \langle n+L-1 \rangle_D + 1 \\ x_{ji,G} & \text{otherwise} \end{cases} \tag{32}$$

is formed where the acute brackets $\langle\ \rangle_D$ denote the modulo function with modulus D. The starting index n in (32) is a randomly chosen integer from the interval [1,D]. The integer L is also drawn from the interval [1,D] with the probability Pr(L=ν) = $(CR)^{\nu}$. CR, the so called crossover probability, is taken from the interval [0, 1] and constitutes a control variable in the design process. The random decisions for both n and L are made anew for each newly generated vector $\underline{u}_{i,G+1}$.

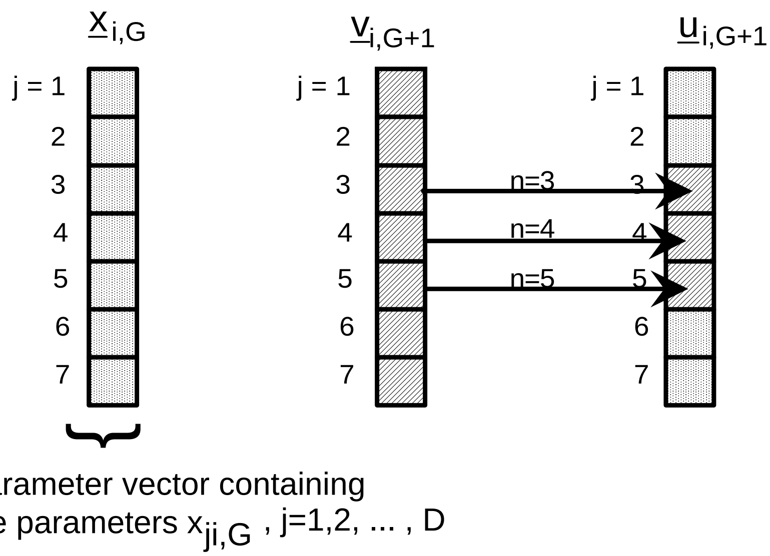

Fig. 3: Illustration of the crossover process for D=7, n=3 and L=3.

In order to decide whether the new vector $\underline{u}_{i,G+1}$ shall become a population member of generation G+1, it will be compared to $\underline{x}_{i,G}$. If vector $\underline{u}_{i,G+1}$ yields a smaller objective function value than $\underline{x}_{i,G}$, $\underline{x}_{i,G+1}$ is set to $\underline{u}_{i,G+1}$, otherwise the old value $\underline{x}_{i,G}$ is retained.

5. Design Results

The optimization task stated above was undertaken using the initial settings:

NP = 300

F = 0.85

CR = 1.

and $a_0 = 2^{-8} = 0.00390625$

The elements of all parameters were initialized with randomly chosen real values between 0 and 1. In order to properly detect the maximum in eqs. (22) and (23) all functions were sampled at 100 equidistant points in the pass band. In the transition band and stop band only 20 points were used to increase computational speed. No particular effort was made to get fastest possible convergence. With the above settings the filter was designed after 1146 generations which required 344100 evaluations of the objective function. Overall computing time was several hours on a 486DX computer with 50MHz clock frequency. The resulting magnitude and group delay response is depicted in Figs. 4 and 5. The corresponding parameter values are:

x_1 = 1.620493889	x_2 = 1.006124616	x_3 = 1.016987443
x_4 = 2.498671532	x_5 = 1.919012547	x_6 = 0.2243566662
x_7 = 0.3745155931	x_8 = 0.4304945767	x_9 = 0.02474720217

$x_{10} =$ 0.1109348238 $x_{11} =$ 0.6369678974 $x_{12} =$ 0.4702593982
$x_{13} =$ 0.408888042 $x_{14} =$ 0.8722907901 $x_{15} =$ 0.2370584458
$x_{16} =$ 0.125761658 $x_{17} =$ 0.05192748457 $x_{18} =$ 0.3109594584

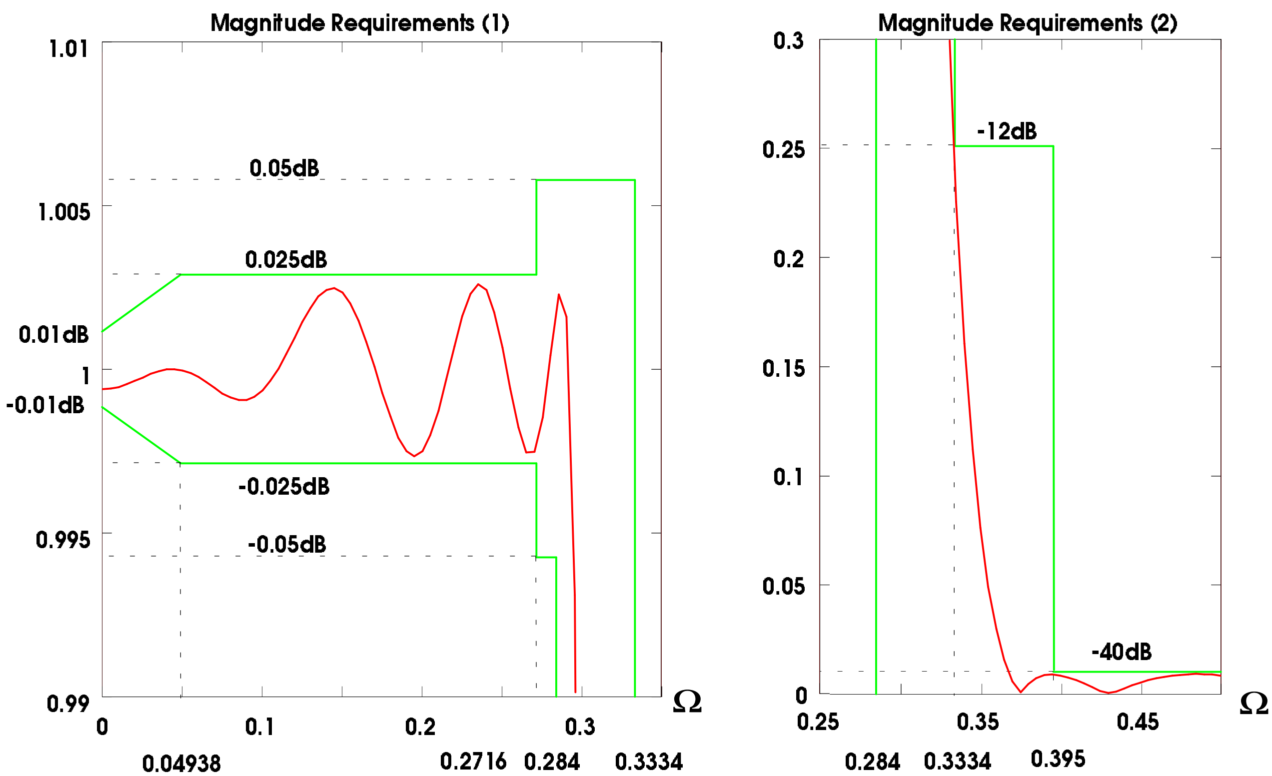

Fig. 4: Magnitude response of the filter design solution for a_0 = 0.00390625.

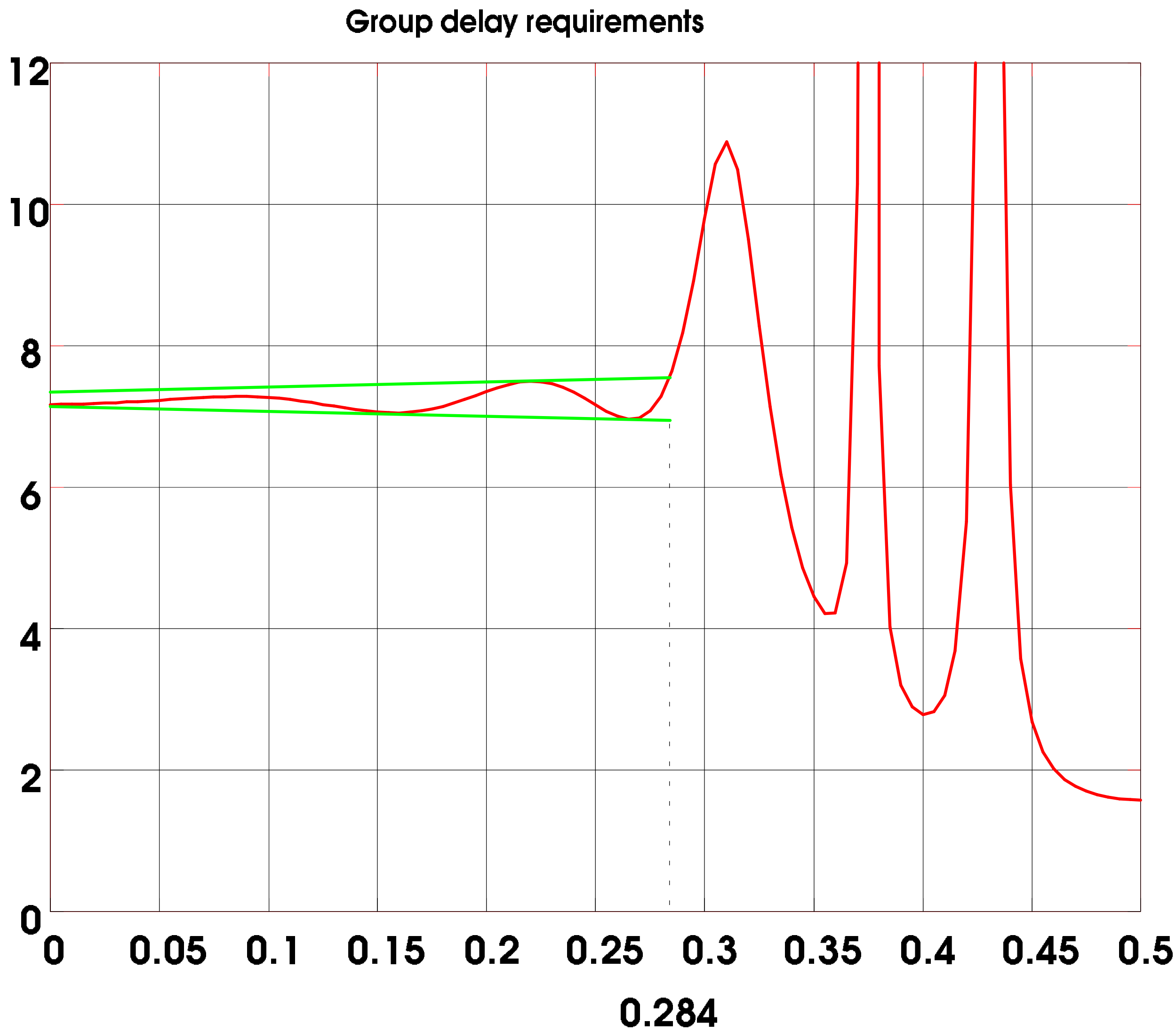

Fig. 5: Group delay response of the filter design solution for $a_0 = 0.00390625$.

The value 2^{-8} for a_0 can be implemented by a simple shift operation and doesn't require a real multiplication. Nevertheless the filter design can be performed for many different values of a_0. Another design example is given below. The setings were

NP = 300

F = 0.85

CR = 1.

and a_0 = 0.01

In this case DE needed 930 generations and 279300 function evaluations. The corresponding parameter values resulting from the optimization are

x_1 =	1.70534277	x_2 =	1.019881606	x_3 =	1.558364391
x_4 =	1.001873851	x_5 =	1.829733968	x_6 =	0.1320674717
x_7 =	0.3776784241	x_8 =	0.2309984416	x_9 =	0.4321155548
x_{10} =	0.04620760679	x_{11} =	0.8493334651	x_{12} =	0.3932341039
x_{13} =	0.6438843012	x_{14} =	0.5428563952	x_{15} =	0.3089904487
x_{16} =	0.01219726913	x_{17} =	0.2359268814	x_{18} =	0.124328509

The magnitude an group delay responses are shown in Figs. 6 and 7.

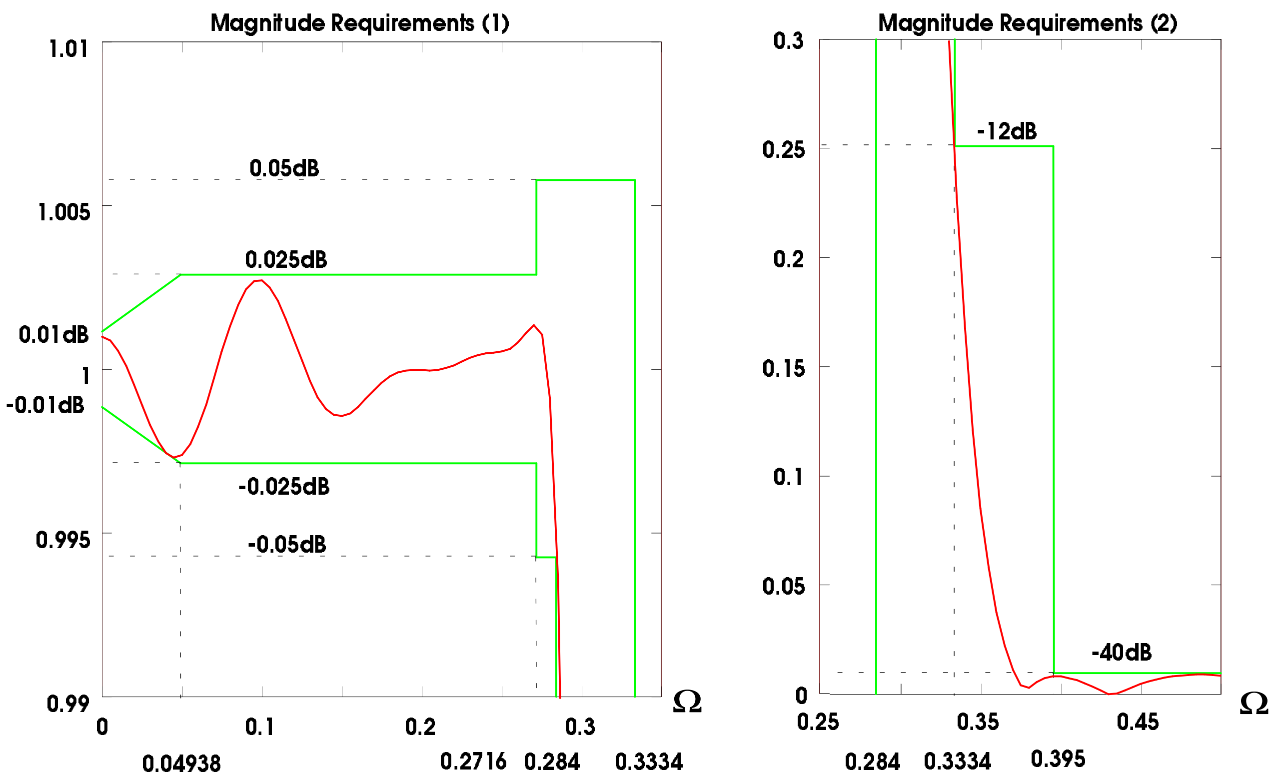

Fig. 6: Magnitude response of the filter design solution for a_0 = 0.01.

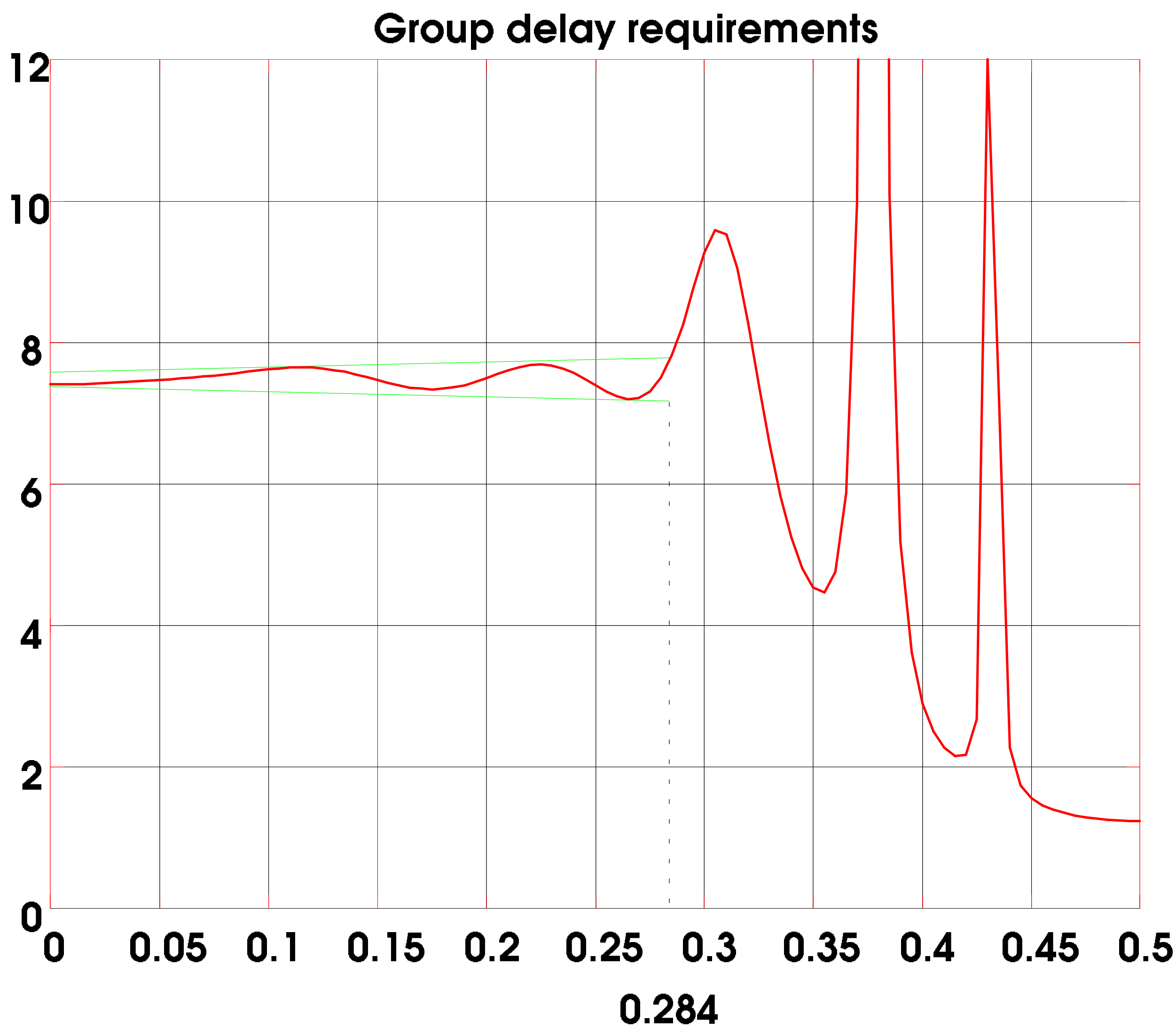

Fig. 7: Group delay response of the filter design solution for $a_0 = 0.01$.

6. Conclusion

It has been demonstrated that it is feasible to design an 18 parameter IIR-filter according to [3] with requirements for both magnitude response and group delay by employing the general purpose minimization method of Differential Evolution (DE). In general the above design problem is a difficult one and requires considerable expertise to be solved successfully. By using DE, however, just basic knowledge about digital filters is required so that no experts are needed for the design task. DE has the additional advantage of being easy to understand, simple to implement and easy to work with so that DE can be used for a wide variety of design and optimization tasks.

7. References

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