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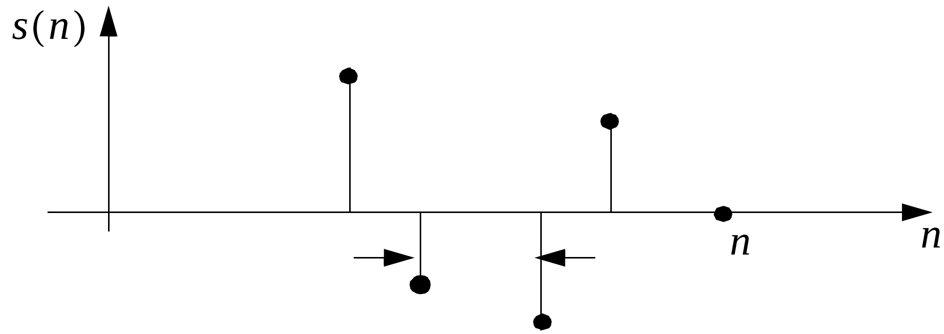
# Digital Signal Processing

# **Lecture 6**

Consider the problem of linear prediction.



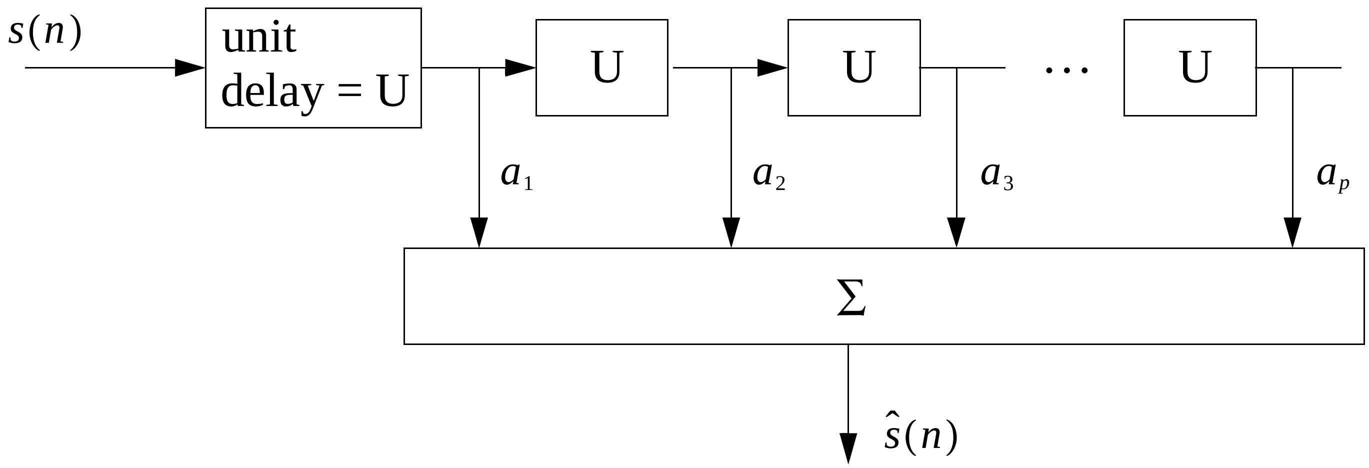

Given a sequence $s(n-p), s(n+1-\rho), s(n+2-p)\ldots$

$s(n-1)$

Predict $\hat{s}(n)$ to be the linear, weighted sum of the above sequence.

$$\hat{s}(n) = \sum_{k-1}^{p} a_k s(n-k) a_k \qquad \text{———} \quad (1)$$

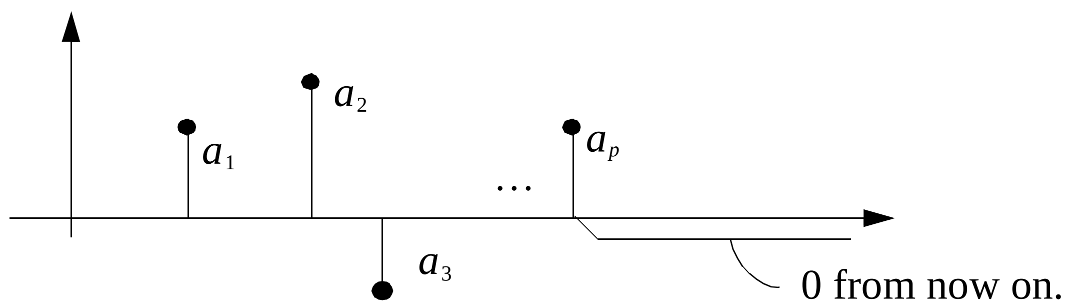
$$\hat{s}(n) = a_1 s(n-1) + a_2 s(n-2) + \ldots a_k s(n-k)$$




If $s(n)$ is a unit pulse - $s(n) = 1$ for n=0

$= 0$ otherwise

$\hat{s}(n) =$




Thus, equation (1) is a Finite Impulse Response (FIR) fiter.

# Linear Recursion.

$$s(n) = e(n) + a_1 s(n-1) + a_2 s(n-2) + \ldots a_p s(n-1p)$$

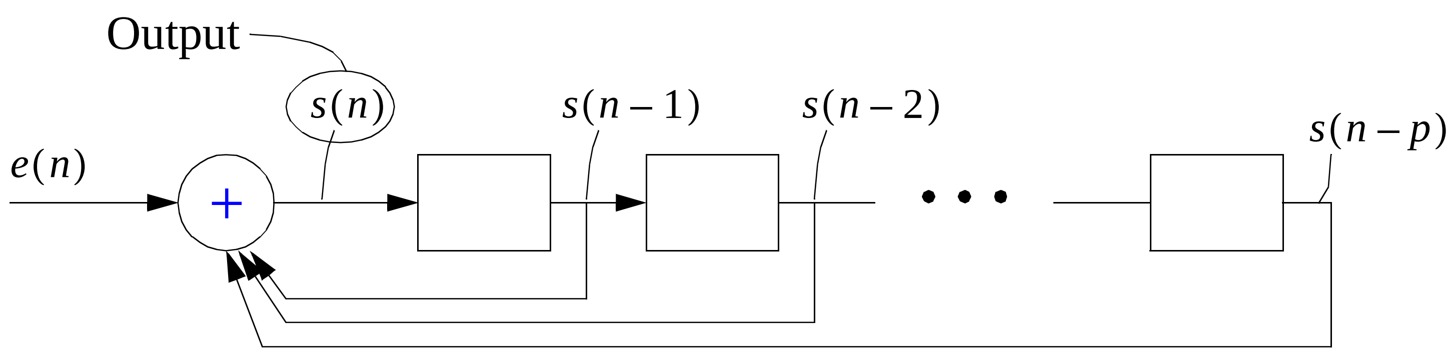
$$e(n) = s(n) - \tilde{s}(n) = s(n) - \sum_{k=1}^{p} a_k s(n-k)$$

$$= \sum_{k=0}^{p} a_k s(n-k) \qquad a_k = 1$$

$$E(z) = \sum_{k=0}^{p} a_k z^{-k} S(z)$$

When $S(z) = z$ - transform of $s(n)$ or $S(z) = \sum_{n=0}^{\infty} z^{-n} s(n)$

$$\therefore S(z) = \frac{E(z)}{\sum_{k=0}^{p} a_k z^{-k}}$$




If you know e(n) perfectly, you can exactly reproduce s(n).

So, LPC methods look for various ways of creating a useful error signal.

How good is the prediction?

Define the error signal.

$$e(n) = s(n) - \hat{s}(n)$$

If we know the error signal, we can construct $s(n)$ perfectly.

$$e(n) = s(n) - a_1 s(n-1) - a_2 s(n-2) - \ldots - a_p s(n-p)$$

OR, $$s(n) = a_1 s(n-1) + a_2 s(n-2) + \ldots + a_p s(n-p) + e(n) \qquad (2)$$

$s(n)$ is recursive ; its value is a function of previous values.

Let's take the simplest case of $p = 1$

$$s(n) = a_1 s(n-1) + e(n)$$

Let's $e(n) = unit$ pulse and $s(-1) = q$

Infinite Impulse Response Filter } IIR

$$\begin{cases} s(0) = a_1 q + 1 \\ s(1) = a_1 s(0) = a_1^2 q + a_1 \\ s(2) = a_1 s(1) = a_1^3 q + a_1^2 \\ \vdots \\ s(m) = a_1 s(m-1) = a_1^{m+1} q + a_1^m \end{cases}$$

If $a_1 < 1, s(m) \underset{m \to \infty}{\to} 0$ stable filter

But if $a_1 > 1, s(m) \underset{m \to \infty}{\to} \infty$ unstable filter

Note : FIR filters are never unstable.

Up to now, we have stayed in the time domain. But samples have a spectrum, just as continuous signals do. To deal with this, we introduce the z-transform, which will allow us to picture what happens when a sampled sinusoid is applied to a discrete linear system.

Definition

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

Where z is a complex variable.

but

$$\hat{X}(z) = \sum_{n=0}^{\infty} x(n-k) z^{-n}$$

$$= z^{-k} \sum_{m=-k}^{\infty} x(m) z^{-m}$$

But $m = n - k$

$n = m + k$

When $n = 0, m = -k$

When $n = \infty, m = \infty$

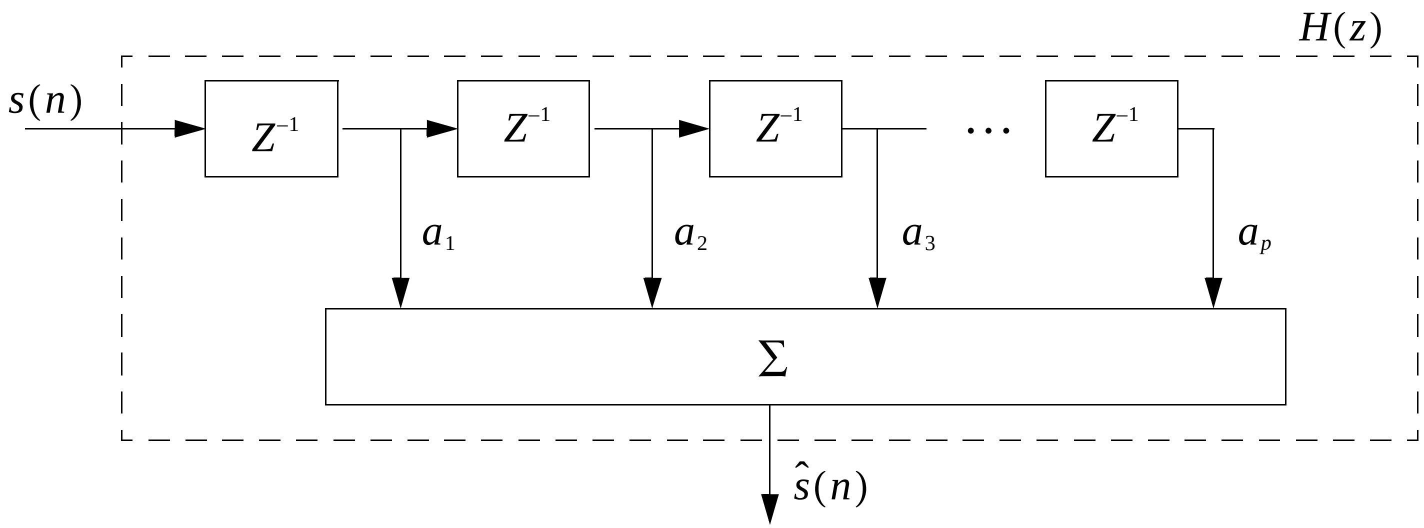
IF $x(m) = 0$ for $m < 0$

$$\hat{X}(z) = z^{-k} \sum_{m=0}^{\infty} x(m) z^{-m} = z^{-k} X(z)$$

We can apply this to equation (1).

$$\hat{S}(z) = (a_1 z^{-1} + a_2 z^{-2} + \ldots + a_k z^{-p}) = z^{-k} X(z)$$

This equation can be represented in terms of z.

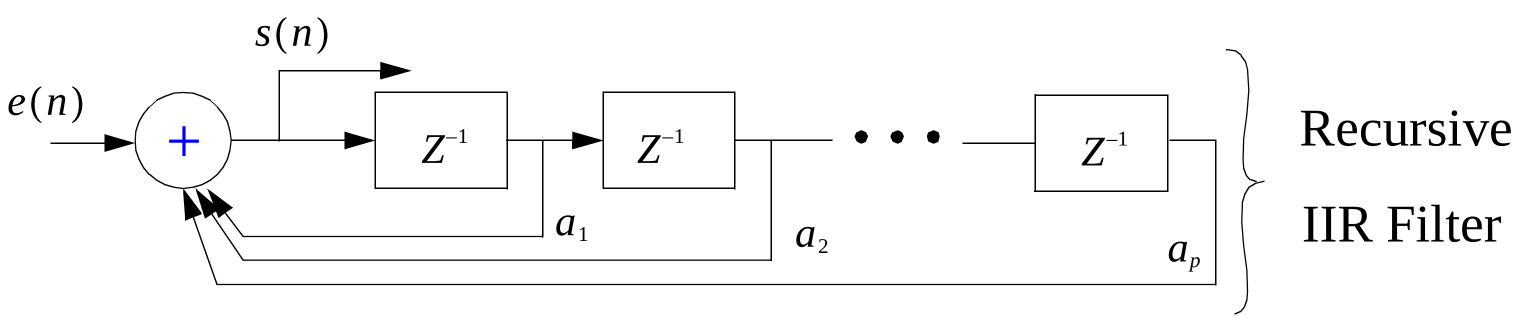


Lets take the z-transform of equation (2).

$$S(z) = (a_1 z^{-1} + a_2 z^{-2} + \ldots + a_p z^{-p}) S(z) + E(z)$$

$$S(z) = \frac{E(z)}{1 - a_1 z^{-1} - a_2 z^{-2} - \ldots - a_p z^{-p}}$$

$$= \frac{E(z)}{1 - H(z)}$$



So, the essence of linear prediction is :

1. Find a set of $a_k$'s that minimize the "complexity" of the error signal.
2. Cleverly code the error signal.
3. Reconstruct $s(n)$ with the about network.

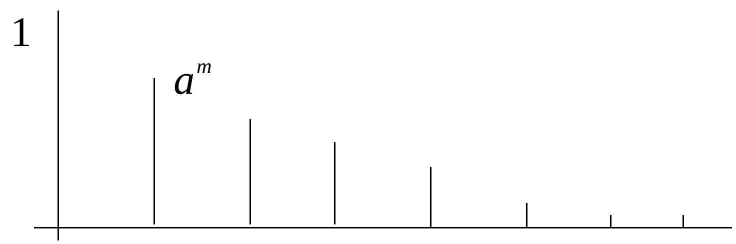
## Convolution - Filtering

If $h(n)$ is the response of a discrete linear system to a unit pulse,

the response to a signal $x(n)$ is

$$y(n) = \sum_{m=0}^{n} x(n)h(n-m) = \sum_{m=0}^{n} x(n-m)h(m)$$

Example ; $h(m) = a^m$ for $m = 0, 1, 2\ldots, a < 1$

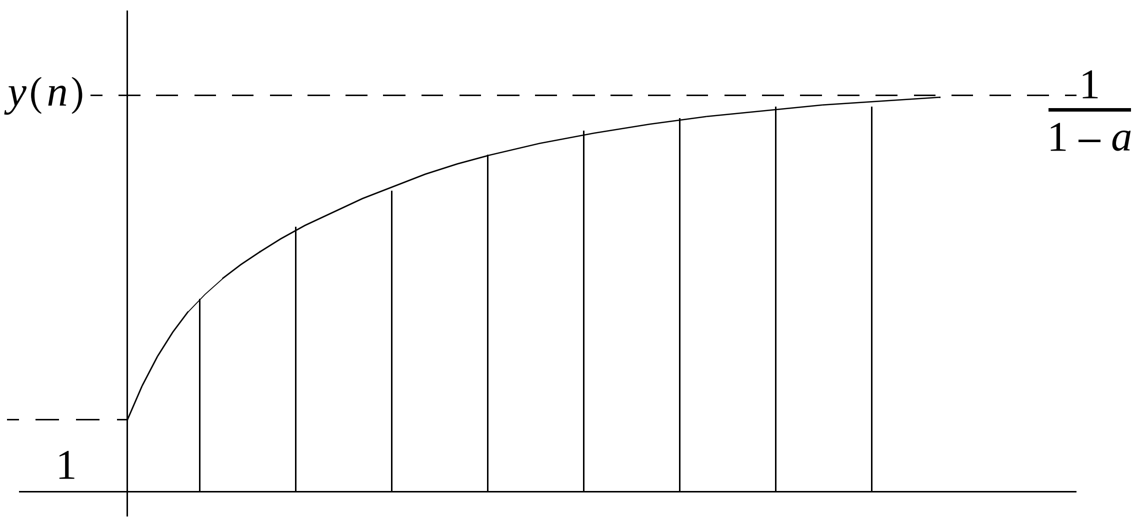


$$y(n) = x(0)a^n + x(1)a^{n-1} + \ldots + x(n)a^0$$

But $x(m)$ be unit step function. $x(m) = 1$ for $m \geq 0$, zero otherwise

$$y(n) = \sum_{m=1}^{n} a^m = \frac{1-a^{n+1}}{1-a} \quad ; \quad n = 0, y(n) = 1$$

$$n \to \infty \qquad y(n) \to \frac{1}{1-a}$$



Now let's do it by z-transform.

$$Y(z) = kz^{-1}Y(z) + X(z)$$

$$Y(z) = \frac{X(z)}{1 - kz^{-1}}$$

$$X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$$

$$y(n) = \frac{1}{2\pi j}\oint Y(z) z^{n-1} dz$$

$$= \frac{1}{2\pi j}\oint \frac{z^{n-1} dz}{(1 - kz^{-1})(1 - z^{-1})}$$

Reduce method of solving integral.

$$\frac{1}{(1-kz^{-1})(1-z^{-1})} = \frac{A}{1-kz^{-1}} + \frac{B}{1-z^{-1}} = \frac{A - Az^{-1} + B - kBz^{-1}}{(1-kz^{-1})(1-z^{-1})}$$

$$B = \frac{1}{1-k}, A = -\frac{k}{1-k}$$ so $$A + kB = 0, A + B = 1$$

$$A = -kB, \quad B(1-k) = 1, \quad B = \frac{1}{1-k}$$

$$\frac{1}{(1-kz^{-1})(1-z^{-1})} = \frac{1}{1-k}\left[\frac{1}{1-z^{-1}} - \frac{k}{1-kz^{-1}}\right]$$

$$A = 1 - B + 1 - k = 1$$

$$1 - \frac{1}{1-k} = 1 - 1$$

Residue =1, Residue =-k

Residue of

$$\frac{1}{1-kz^{-1}} \Rightarrow ,k^n$$

Solution is $\frac{1}{1-k}[1-k^{n+1}]$