What does it mean for a language to be **regular**?
Is the following language regular?

$L = \{ w \mid w \text{ has the same number of 01s as 10s} \}$

$\Sigma = \{ 0, 1 \}$

Is the following language regular?

$L = \{ w \mid w \text{ has the same number of 0s and 1s} \}$

$\Sigma = \{ 0, 1 \}$
How do we prove that a language is not regular?

1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.

Is the following language regular?

$L = \{a^n b^n\}$
How to show a language $L$ is not regular:
1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.
Let's look at strings $s$, $|s| = 4$

What is true about the automaton for all strings of length 4 it reads?

What is the shortest string that will cause a repeated state?

Suppose DFA $M$ has $|Q|$ states. Any string $s$, $|s| \geq |Q|$ will cause $M$ to repeat a state.

What does it mean to repeat a state?
Input: 10
State Sequence: ABA

Also accepted:
1010
101010

Input: 010
State Sequence: ABBC

Also accepted:
01*0
Generalized process:
1. Pick a string \( s \) of length \(|Q|\).
2. Find where it repeats a state.
3. Repeat that part of the string.

\[ s = xyz, \text{ where } y \text{ is the repeating part.} \]

How to show a language \( L \) is not regular:
1. Identify some property \( P \) that is true for all regular languages.
2. Assume \( P \) holds true for language \( L \).
3. Obtain a contradiction, thereby showing \( L \) is not regular.

(Draft 1)
Property: Every regular language with a DFA of \(|Q|\) states has a string \( s \) of length \(|Q|\) where \( s = xyz \) and \( y \) can be repeated.

(Draft 2)
Property: Every regular language with a DFA of \(|Q|\) states has a string \( s \) of length \(|Q|\) where \( s = xyz \) and \( y \) can be repeated and \(|y| > 0\).

(Draft 3)
Property: Every regular language with a DFA of \(|Q|\) states has a string \( s \) of length \(|Q|\) where \( s = xyz \) and \( y \) can be repeated, \(|y| > 0\), and \(|xy| \leq |Q|\).
Pumping Lemma

For every regular language \( L \) there exists some integer \( p \) where for every string \( s \) in \( L \) of length at least \( p \), \( s = xyz \) and \( y \) can be repeated, \(|y| > 0\), and \(|xy| \leq p|.

How to show a language \( L \) is not regular:
1. Identify some property \( P \) that is true for all regular languages.
2. Assume \( P \) holds true for language \( L \).
3. Obtain a contradiction, thereby showing \( L \) is not regular.

Is the following language regular?

\[
L = \{0^n1^n \mid n \geq 0\}
\]

\[\Sigma = \{0, 1\}\]

1. Identify some property \( P \) that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string \( s \) of length \( p \) for which we can apply the pumping lemma.

2. Assume \( P \) holds true for language \( L \).
Is the following language regular?
\[ L = \{0^n1^n \mid n \geq 0\} \]

1. Identify some property P that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some
string s of length p for which we can apply the pumping lemma.

2. Assume P holds true for language L.

Assume L is regular and thus has a DFA
and is pumpable.

3. Obtain a contradiction, thereby showing L is not regular.

Let \( s = 0^p1^p \) \( s \in L \)
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \( s = 0^p1^p \ s \in L \)

The pumping lemma guarantees:

\[ s = xyz \ xy^*z \in L \]

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3 CASES

1. \( y \in \{0^*\} \)
Is the following language regular?

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3 CASES

1. \( y \in \{0^*\} \)
2. \( y \in \{1^*\} \)

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Let \( s = 0^p1^p \ s \in L \)

The pumping lemma guarantees:

\[ s = xyz \ xy^*z \in L \]

3 CASES

1. \( y \in \{0^*\} \)
2. \( y \in \{1^*\} \)
3. \( y \in \{0^i1^j\} \)

Impossible because repeating \( y \) would produce more 0s than 1s.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

\[ s = xyz \quad xy^*z \in L \]

3 CASES

1. \( y \in \{0^*\} \)  Impossible because repeating \( y \) would produce more 0s than 1s.

2. \( y \in \{1^*\} \)  Impossible because repeating \( y \) would produce more 1s than 0s.

3. \( y \in \{0^i1^j\} \)  Impossible because repeating \( y \) would mis-order 1s and 0s.

Therefore

\[ L = \{0^n1^n \mid n \geq 0\} \]

is NOT regular.
Is the following language regular?
\[ L = \{ w \mid w \text{ has the same number of } 0\text{s and } 1\text{s} \} \]
\[ \Sigma = \{ 0, 1 \} \]

Pumping Lemma
For every regular language \( L \) there exists some integer \( p \) where for every string \( s \) in \( L \) of length at least \( p \), \( s = xyz \) and \( y \) can be repeated, \( |y| > 0 \), and \( |xy| \leq p \).
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2. Assume \( P \) holds true for language \( L \).

Assume \( L \) is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing \( L \) is not regular.
\[ s = 0^p1^p \]
\[ |xy| \leq p \implies y \in \{ 0^* \} \implies xy^2z \notin L \]
Is the following language regular?

\[ L = \{ ww^R \} \]
\[ \Sigma = \{ 0, 1 \} \]

**Pumping Lemma**

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Is the following language regular?

\[ L = \{ ww^R \} \]

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   There exists some DFA that recognizes any regular language and accepts some string \( s \) of length \( p \) for which we can apply the pumping lemma.

2. Assume \( P \) holds true for language \( L \).

   Assume \( L \) is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing \( L \) is not regular.

   \[ s = 0^p110^p \]
   \[ |xy| \leq p, |y| > 0 \implies y \in \{0^+\} \implies xy^0z \notin L \]
Is the following language regular?
\[ L = \{0^i1^j \mid i > j \} \]
\[ \Sigma = \{0, 1\} \]

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For every regular language \( L \) there exists some integer \( p \) where for every string \( s \) in \( L \) of length at least \( p \), \( s = xyz \) and \( y \) can be repeated, \( |y| > 0 \), and \( |xy| \leq p \).
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There exists some DFA that recognizes any regular language and accepts some string \( s \) of length \( p \) for which we can apply the pumping lemma.

2. Assume \( P \) holds true for language \( L \).

Assume \( L \) is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing \( L \) is not regular.

\[ s = 0^{p+1}1^p \]

\[ |xy| \leq p, |y| > 0 \implies y \in \{0^+\} \implies xy^0z \notin L \]
Is the following language regular?

\[ L = \{0^i1^j0^k \mid i > 10 > j > k > 0\} \]

\[ \Sigma = \{0, 1\} \]

\[ R = 0^+0^{10}((1^9(0^8 \cup 0^7 \cup \ldots \cup 0^1)) \cup (1^8(0^7 \cup 0^6 \cup \ldots \cup 0^1)) \cup \ldots \cup (1^2(0^1))) \]

**YES**

Reading: Sipser 1.4