Due: November 2, 2009 in class or 1:00 pm (via email to cs361-hw@cs.williams.edu)

1. Closure (15 points). Prove that $RE$ is a) closed under union, b) closed under intersection, and c) closed under Kleene star.

2. Queues (10 points). Prove that $L \in RE$ if and only if there exists some queue automaton that recognizes $L$.

3. CFL/REG (10 points). Prove that the following language $L$ is undecidable:
   $$L = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \in \text{CFL and } L(M) \notin \text{REG}\}.$$  

4. RE/coRE (10 points). Prove that the following language $L$ is neither in $RE$ nor in $coRE$:
   $$L = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } M_1(\epsilon) \text{ halts and } M_2(\epsilon) \text{ loops}\}.$$  

5. Verifiers (10 points). Prove that $L \in RE$ if and only if $L$ is verifiable.

6. Separability (20 points). Let $A, B, D$ be languages. We say that $D$ separates $A$ from $B$ if $A \subseteq D$ and $D \cap B = \emptyset$. We say that a language class $C$ is separable if for every $A, B \in C$ where $A \cap B = \emptyset$ there exists a decidable language $D$ such that $D$ separates $A$ from $B$. Prove that $coRE$ is separable.

7. Separability (25 points). Prove that $RE$ is not separable.