OPTIMAL RESERVOIR OPERATION USING DIFFERENTIAL EVOLUTION

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ABSTRACT

In the present study an evolutionary based optimization algorithm Differential Evolution (DE) is applied to a case study of Bisalpur project, Rajasthan, India. The objective of DE based planning model is to determine suitable cropping pattern which yields maximum net benefits. Ten different strategies (variations) of DE are analyzed with various population sizes, crossover constants and weighting factors. Results of DE are compared with solution of Linear Programming (LP). Minimum and maximum CPU time that was elapsed is also analyzed. It is concluded that DE/rand-to-best/1/bin is the best strategy for the planning problem with maximum net benefits of 95.1903 crores of rupees taking minimum CPU time of 2.844 seconds. The present study can be extended to similar situations with suitable modifications.

INTRODUCTION

Need for efficient integrated management of an irrigation system is keenly felt due to growing demand for agricultural products, the escalating costs of supplying water to farmer's fields and stochastic nature of water resources (Raju and Kumar, 2003). Due to dwindling supply of water the profit conscious irrigators wish to so allocate the water as to maximize the net benefits with competing alternative crops. Investor's choice is further complicated by the fact that the allocation of water is required to be optimized over time, among the crops and also among the competing units of the same crop simultaneously. This necessitates integration of mathematical models with irrigation management aspects for better planning (Raju, 1995). In this paper, an irrigation planning model is formulated keeping net benefits as the objective function.

DIFFERENTIAL EVOLUTION (DE)

Differential Evolution (DE) is an evolutionary optimization technique, which is simple, significantly faster and robust at numerical optimization and likely chances of finding true global optimum. DE (Price and Storn, 1997) is an improved version of Genetic Algorithms (Goldberg, 1989) for faster optimization. The principal difference between Genetic Algorithms and Differential Evolution is that Genetic Algorithms rely on crossover, a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions, while evolutionary strategies use mutation as the primary search mechanism (Godfrey and Babu, 2004). Differential Evolution (DE) uses a non uniform crossover that can take child vector parameters from one parent more often than it does from others. By using components of existing population members to construct trial vectors, recombination efficiently shuffles information about successful combinations, enabling the search for an optimum to focus on the most promising area of solution space.

If the fitness of the trial vector turns out to be less than or equal to that of its parent target, the trial vector replaces the target as the population vector of the next generation. DE is used to find the optimum values for network parameters such that the network learning time is reduced and recognition accuracy is increased.

Parameters in real-world problems generally exhibit restricted ranges over which it is sensible to search for a solution. Before DE starts optimizing a function, these parameter limits should be established. Once limits have been set, each parameter in every primary array vector is initialized with a uniformly distributed random value from within its allowed range. To determine and preserve the resulting initial population, each primary array vector is evaluated and the results are stored in the array, cost [ ].

A more convenient source of appropriately scaled perturbations is the population itself. Every pair of vectors (x_a, x_b) defines a vector differential: $(x_a - x_b)$. When x_a and x_b are chosen randomly, their weighted difference can be used to perturb another vector, x_c. This process, which can be expressed mathematically as

$$x_c' = x_c + F(x_a - x_b) \tag{1}$$

The weighting factor F is a user-supplied constant in the range (0<F≤1.2) (Price and Storn, 1997). The upper limit 1.2 is determined empirically. The optimal value of F in most functions lies in the range 0.4 to 1.0. By mutating vectors, DE ensures that the solution space will be efficiently searched in each dimension. For example, if the population becomes compact in one dimension but remains widely dispersed along another, the differentials sampled from it will be small in one dimension, yet large in the other.

Once in every generation, each primary array vector, x_i for $i = 0, 1,$ is targeted for recombination with a vector like x_c' to produce a trial vector x_t. Thus, the trial vector is the child of two parents: a noisy random vector and the target vector against which it must compete. Which parent contributes which trial vector parameter is determined by a series of binomial experiments. Each experiment, whose outcome is either success or failure, is mediated by the crossover constant CR, where 0 ≤ CR ≤1. Starting at a randomly selected parameter, the source of each trial vector parameter is determined by comparing CR to a uniformly distributed random number from within the interval [0,1]. If the random number is greater than CR, the trial vector gets its parameter from the target x_i, otherwise, the parameter comes from the noisy random vector x_c'.

DE does not use proportional selection, ranking, or even an annealing criterion that would allow occasional uphill moves. Instead, the cost of each trial vector is compared to that of its parent target vector. The vector with the lower cost is rewarded by being allowed to advance to the secondary array. In addition, if the trial vector wins, its cost is stored in cost [i]. After each primary array vector has been a target for mutation, recombination, and selection, array pointers are swapped so that the roles of the two arrays are reversed. Thus, vectors in what was the secondary array become targets for transformation, while the former primary array now awaits winners of the next generation's competitions.

The idea behind DE is a scheme for generating the trial vectors. Basically DE adds the weighted difference the two population vectors to a third vector. Price and Storn (1997) gave the working principle of DE with single strategy. Later on, they suggested ten different strategies namely, DE/rand/1/bin, DE/best/1/bin, DE/best/2/bin, DE/rand/2/bin, DE/randtobest/1/bin, DE/rand/1/exp, DE/best/1/exp, DE/best/2/exp, DE/rand/2/exp, DE/randtobest/1/exp (Price and Storn, 2004). DE/x/y/z indicates DE for Differential Evolution, x is a string which denotes the vector to be perturbed, y denotes the number of difference vectors taken for perturbation of x and z is the crossover method. A strategy that works out to be best for a given problem may not work well when applied for a different problem. Also, the strategy and key parameters adopted for a problem are to be determined by extensive sensitivity analysis.

CASE STUDY

In the present study, Differential Evolution based irrigation planning model is formulated and applied to a case study of Bisalpur project, Rajasthan, India, to evolve a suitable optimum cropping pattern to yield maximum net benefits while meeting the drinking water requirements.

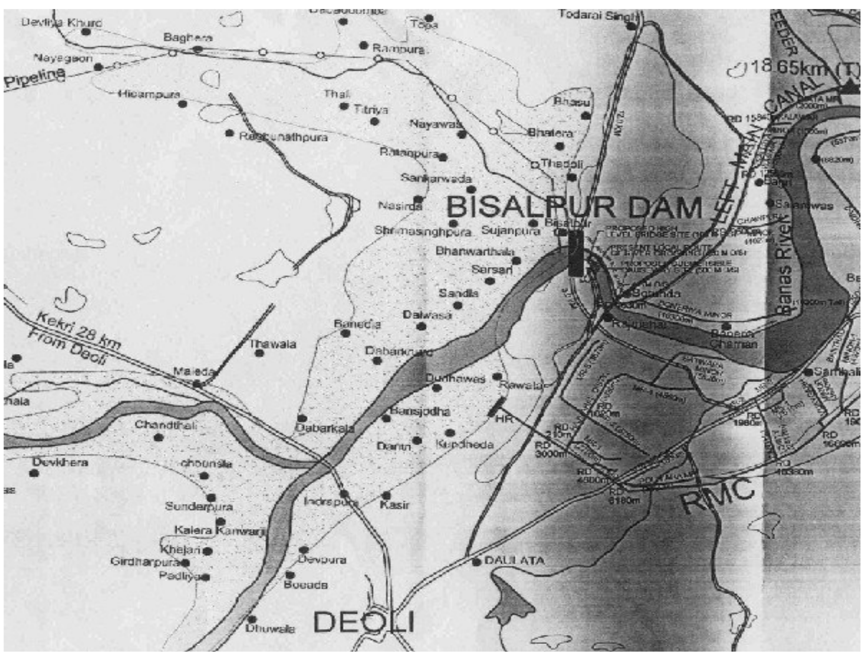

Fig. 1 Location map of Bisalpur Project

Bisalpur project is a major project located on river Banas. The project is meant for irrigation and drinking water. The dam site is located at latitude 25°55' N and longitude 75° 27' E. Two canal systems are originating from the project, namely, left canal (under construction) and right canal having Culturable command areas of 8407 ha, and 68,293 ha. Irrigation intensity in the project is 72%. Fig. 1 presents location map of the project. Gross and live storage capacities for the reservoir are 1100 Mm^3, 896.5 Mm^3 (i.e., 1 $Mm^3 = 10^6$ m^3). Crops in the command area are Jowar, Maize, Groundnut, Soyabeen, Wheat, Barley, Gram, and Mustard. Bisalpur project covers two districts, namely, Tonk and Sawai Madhopur for irrigation and Ajmer, Jaipur and Tonk Districts for drinking water. Total number of villages benefited from the project is 241. Overall efficiency of the project is around 49% (Bisalpur drinking water cum irrigation project report, 1999).

Mathematical Modelling

Mathematical modelling of the objective function and the corresponding constraints is explained below. The net benefits (*BE*) from different crops are to be maximized. These are obtained by subtracting the cost of surface water from gross benefits of crops. Mathematically it can be expressed as

$$BE = \sum_{i=1}^{9} B_i A_i \qquad (2)$$

where i is Crop index [1=Maize (K), 2=Jowar (K) , 3=Ground nut (K) , 4=Soya bean (K), 5=Wheat (R), 6=Gram (R), 7=Barley (R), 8= Coriander (R), 9= Mustard (R), K = Kharif , R = Rabi]; t is Time index (1=January, ..., 12=December). BE = Net benefits from the whole planning region (Indian Rupees); B_i = Net benefits from the crop i (excluding cost of surface water, seeds, fertilizers etc) A_i, = Area of crop i grown in the command area (ha); IR_t = Irrigation releases from reservoir to command area (Mm^3).

The model is subjected to the following constraints

1. Continuity Equation

Reservoir operation includes water transfer, storage, inflow and spillage activities. Water transfer activities consider transport of water from the reservoir to the producing areas through canals to meet the water needs. A monthly continuity equation for the reservoir storage (Mm^3) can be expressed as

$$S_{t+1} = S_t + I_t - IR_t - DW_t - EV_t \qquad for\ t = 1,2,.......12 \qquad (3)$$

where S_{t+1} = End of month reservoir storage in the Bisalpur reservoir (Mm^3); I_t = Monthly net inflows into the reservoir (Mm^3); IR_t = Monthly irrigation releases (Mm^3); DW_t = Monthly drinking water releases (Mm^3); EV_t = Monthly net evaporation volume (Mm^3). Releases for drinking water are made equal to their demands.

The above constraint assumes that the monthly inflows into the reservoir are known with certainty. When stochasticity is incorporated in the inflow terms, the above equation changes to

$$S_{t+1} - S_t + EV_t + IR_t + DW_t \geq I_t^{\alpha} \qquad for \quad t = 1, 2,......,12 \qquad (4)$$

where I_t^{α} is inverse of the cumulative distribution of inflows at reliability level α.

2. Crop Area Restrictions

The total cropped area allocated for different crops in a particular season should be less than or equal to the gross irrigated area (*GIA*).

$\sum_{i} A_i \le GIA$; i=1,2,3,4 Kharif season (5)

$\sum_{i} A_i \le GIA$; i=5,6,7,8,9 Rabi season (6)

3. Crop Water Diversions

Monthly crop water requirements CWR_{it} are obtained from the project reports. In the absence of any crop activity CWR_{it} is taken as zero. Irrigation water releases from the reservoir must satisfy the irrigation demands of the command area.

$$IR_t - \sum_{i=1}^{9} CWR_{it} A_i = 0 \qquad for \quad t = 1, 2, ..., 12 \qquad (7)$$

where CWR_{it}= Crop water requirements for crop i in month t (in depth units, m).

4. Canal Capacity Restrictions

Total water releases from reservoir (drinking and irrigation) cannot exceed the canal capacity.

$$IR_t + DW_t \le CC \qquad for \quad t = 1, 2, ..., 12 \qquad (8)$$

where CC = Canal capacity. In the present study, canal capacity is converted into volumetric units, Million cubic meters (Mm^3), to be compatible with releases.

5. Live Storage Restrictions

Reservoir storage volume S_t in any month t must be less than or equal to live storage of the reservoir.

$$S_t \le \text{LS} \qquad \text{t} = 1, 2, ..., 12 \qquad (9)$$

where LS= Live storage of the reservoir (Mm^3)

The other constraints incorporated into the model are crop area restrictions, evaporation loss and drinking water requirements. In the present study fifty percent dependable inflow scenario is considered which amounts to annual inflow of 1196 Mm^3. Only cropping pattern related to right main canal (RMC) is considered as per original project proposal. It is also assumed that drinking water will be supplied through canal network even though laying of pipeline for drinking water is under progress. Annual drinking water demands (after considering overall efficiency) and evaporation losses as per latest estimates are 95.87 Mm^3, 174.14 Mm^3 respectively (Bisalpur drinking water cum irrigation project report, 1999).

RESULTS AND DISCUSSIONS

The above mathematical model is solved using Differential Evolution and compared with Linear Programming (LP) solution (Raju and Kumar, 2003). Total number of constraints and bounds in the model are 49 and 66 respectively. An interactive computer program is developed in C environment which can handle any number of NP, CR and F. Output of the program is stored in a html file. Interested readers can download the output file from http://www.geocities.com/vasan_a/BisalpurDE.html. Penalty function approach is used to convert the constrained problem into unconstrained problem with a high penalty function value of 10^{19} (Deb, 1995). The model is run for different combinations of ten DE strategies, NP (200 to 1000 with an increment of 50), CR (0.4 to 1.0 with an increment of 0.5) and F (0.4 to 1.0 with an increment of 0.5) to determine the optimum value of the objective function. The best combination of NP, CR and F which will yield maximum net benefits (with an accuracy of 10^{-7}) from all the different combinations is chosen for each strategy. These are presented in Table 1. In addition, comparison of CPU time in seconds for each strategy is also performed which is based on PC with PIV 2.4GHz/256MB RAM/40GB HDD. The following observations are made from the analysis of results.

- It is evident from Table 1 that strategy 10, DE/rand-to-best/1/exp with NP=500, CR=0.95 and F=0.50 is the best strategy as it produces maximum net benefits (95.1903 Crores of rupees) with no constraint violation taking minimum CPU time (2.844 seconds).
- On the other hand, strategies 1, 5, 7, 8 and 10 are yielding approximately same optimal values of benefits with minor difference as solution is of the accuracy range of 10^{-7} even though there is small differences in CPU time. Reaching the same optimal values from number of strategies may be due to the sufficient resources that are available to satisfy demands.
- Efforts are also made for comparison of cropping pattern obtained by DE strategy 10 (DE/rand-to-best/1/exp) and Linear Programming (LP) which is shown in Table 2 (Raju and Kumar, 2003). It is observed that the cropping pattern is almost the same with both the methodologies indicating that DE can be used as alternative methodology to LP even in cases where constraints and bounds are more and nonlinear nature of objective function exists.
- It is observed from Fig. 2 that the strategies with exponential crossover are closer to the optimum obtained by LP (with minimum variation) irrespective of the combination of NP, CR and F as compared to binomial crossover.
- Fig. 3, Fig. 4, Fig. 5 and Fig. 6 shows the effect of varying weighting factor F (0.4 to 0.95 amounting to 12 levels), NP values (200 to 1000 amounting to 17 levels) for 4 different strategies, namely, DE/rand/1/bin, DE/best/1/bin, DE/rand/1/exp and DE/best/1/exp (keeping CR=0.95) on net benefits. It is inferred from Fig. 3 that there is decreasing trend of net benefits with increase in values of F for DE/rand/1/bin whereas consistency in benefits is observed in case of DE/best/1/bin (Fig. 4). It is observed from Fig.5 that decrease in benefits is observed in a narrow region. Similarly, benefits are consistent upto F value 0.65 and thereafter these are decreasing (Fig. 6). Finally it is inferred from Figs. 3 to 6 that lower value of F gives a higher chance of convergence to the optimum and there is a gradual decrease in the optimal value as F increases.
- Similarly, it is observed from Fig. 7 and Fig. 8 that benefits are increasing as value of CR is increasing for both strategies DE/rand/1/bin and DE/rand/1/exp for a fixed F value of 0.4. It is inferred that DE is much more sensitive to the choice of F than it is to the choice of CR as there is a considerable variation in benefits when F increases and not the same with the variation of CR.
- It is also observed that benefits are converging to optimum value with CR=0.95 as high values of CR results in a rotationally invariant sampling of the search space and helps in a faster and/or more robust convergence (Godfrey and Babu, 2004).
- Fig. 9 shows the variation of population size NP for 4 different strategies namely, DE/rand/1/bin, DE/best/1/bin, DE/rand/1/exp and DE/best/1/exp for a fixed CR and F value of 0.4. It is observed that the variation of NP does not follow a regular pattern and it needs a trail and error procedure to determine the optimum NP value.

Table 1 Results of DE with all ten strategies (with an accuracy of 10^{-7})

Strategy No.	Strategy	NP	CR	F	Optimal Value (in crores)	Constraint Violation	Time taken* (seconds)
1	DE/rand/1/bin	200	0.95	0.40	95.1901	0	1.843
2	DE/best/1/bin	200	0.95	0.40	97.722	602.55	1.813
3	DE/best/2/bin	250	0.90	0.90	96.0274	379.58	2.594
4	DE/rand/2/bin	350	0.75	0.95	94.9971	282.06	3.860
5	DE/rand-to-best/1/bin	300	0.85	0.50	95.1903	0	3.078
6	DE/rand/1/exp	350	0.95	0.40	95.1832	0	2.000
7	DE/best/1/exp	450	0.95	0.55	95.1903	0	3.157
8	DE/best/2/exp	250	0.95	0.40	95.1902	0	1.375
9	DE/rand/2/exp	850	0.85	0.40	95.1212	0	4.329
10	**DE/rand-to-best/1/exp**	**500**	**0.95**	**0.50**	**95.1903**	**0**	**2.844**

* CPU time on a PC with PIV 2.4GHz/256MB RAM/40GB HDD

Table 2 Cropping Pattern obtained by the two methods DE & LP

S. No.	Crop Name	Crop Area ('00 ha)	
		DE (DE/rand-to-best/1/exp)	LP
1	Maize (K)	44.73	44.73
2	Jowar (K)	93.19	93.190002
3	Groundnut (K)	28.38	28.379999
4	Soyabean (K)	27.72	27.719999
5	Wheat (R)	149.87	149.869995
6	Gram (R)	70.87	70.870003
7	Barley (R)	22.63	22.629999
8	Corainder (R)	11.51	11.51
9	Mustard (R)	50.0	50.0
Net Benefits (Crores of Rupees)		**95.1903389**	**95.19034**

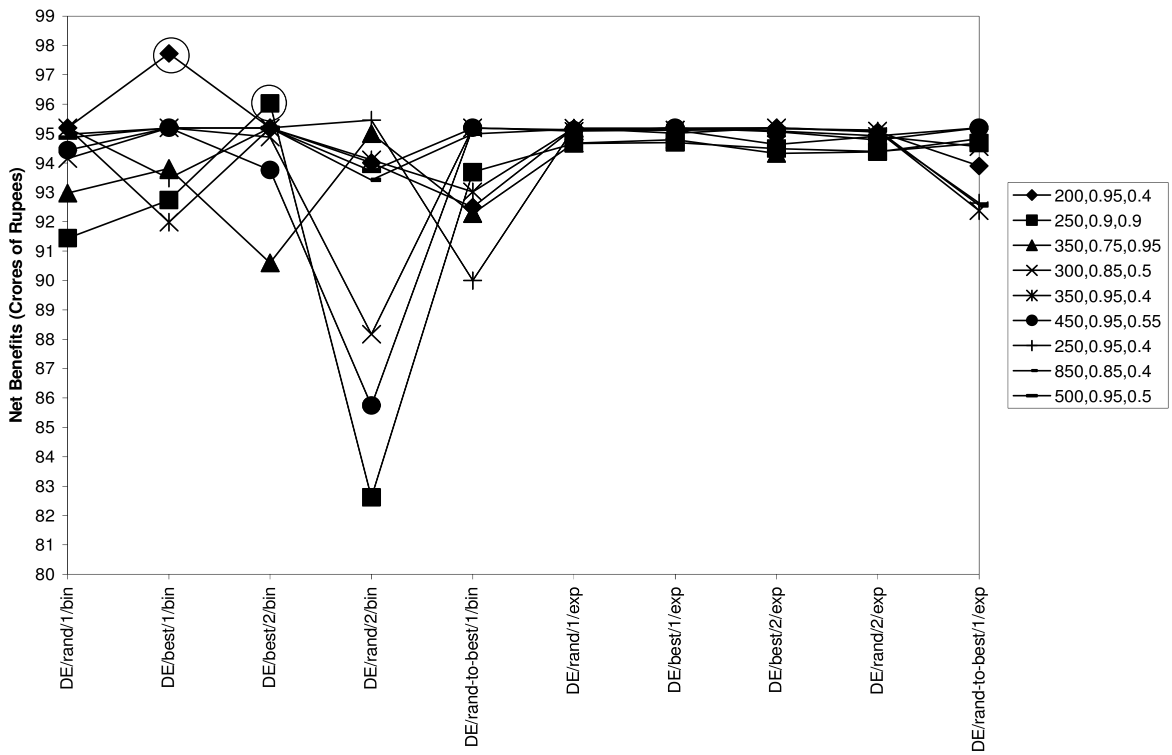

Fig. 2. Strategy variation for a sample set of parameters NP, CR & F

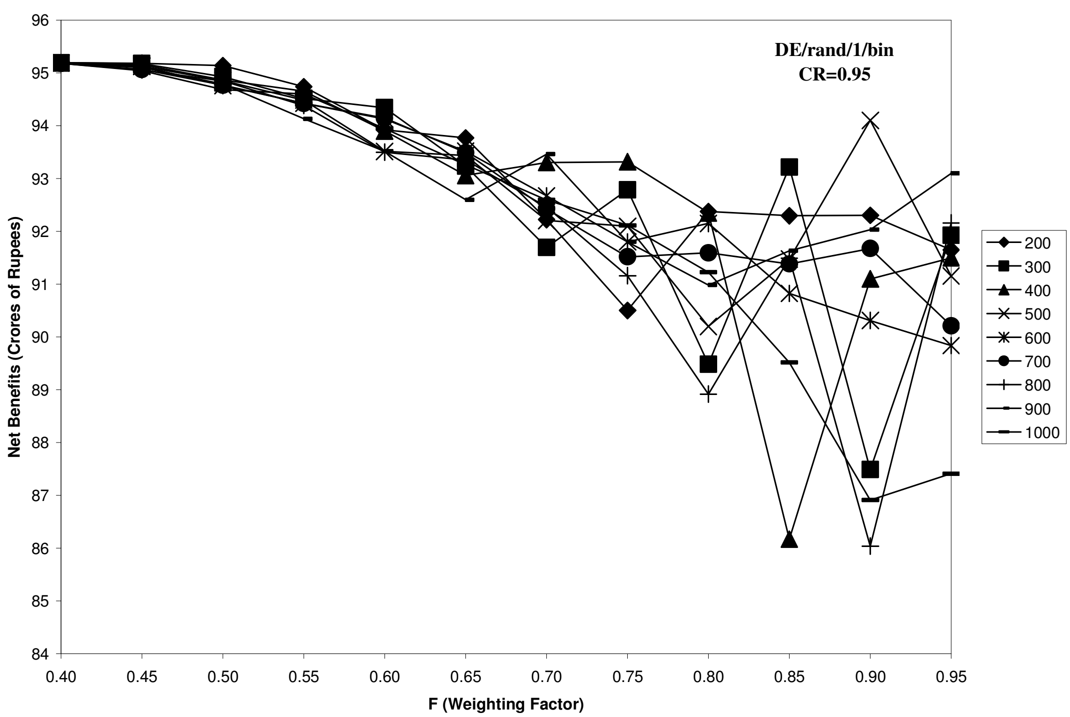

Fig. 3. Variation of weighting factor F for different NP values keeping CR =0.95 for strategy DE/rand/1/bin

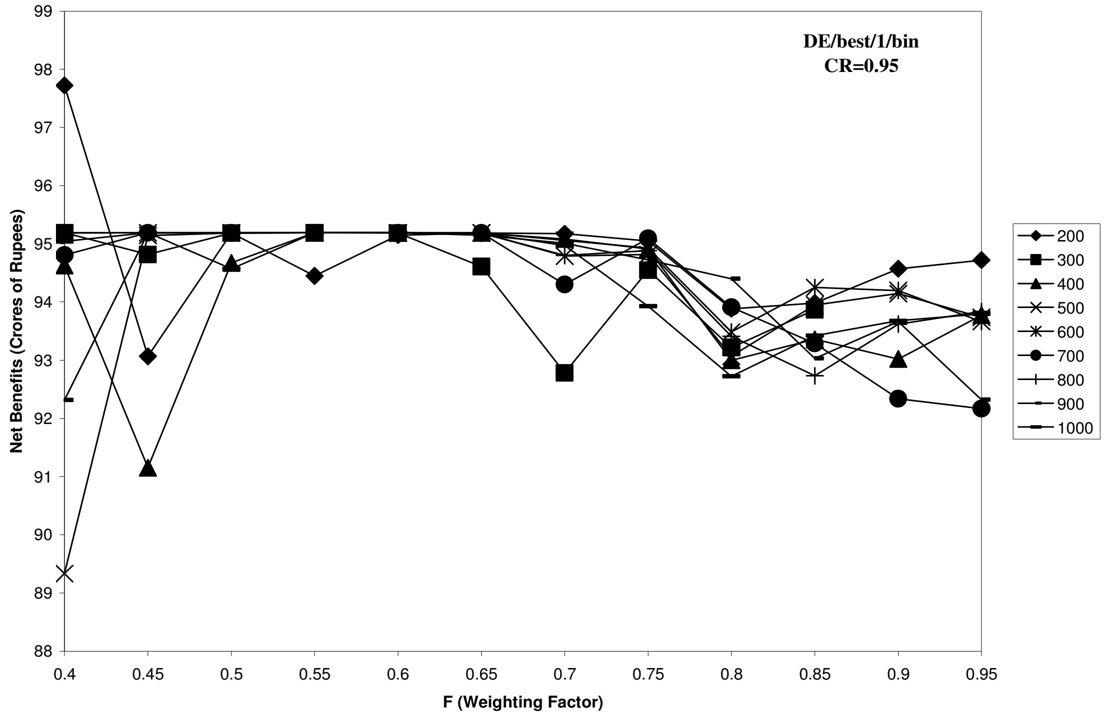

Fig. 4. Variation of weighting factor F for different NP values keeping CR =0.95 for strategy DE/best/1/bin

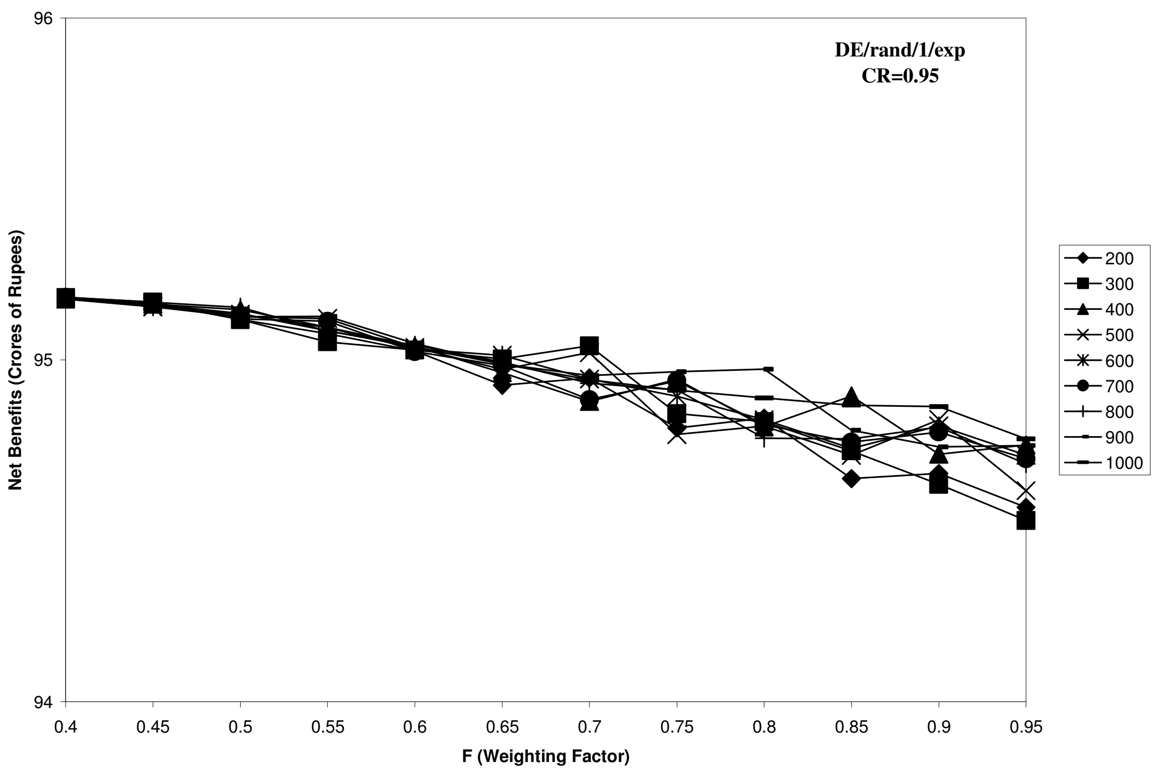

Fig. 5. Variation of weighting factor F for different NP values keeping CR =0.95 for strategy DE/rand/1/exp

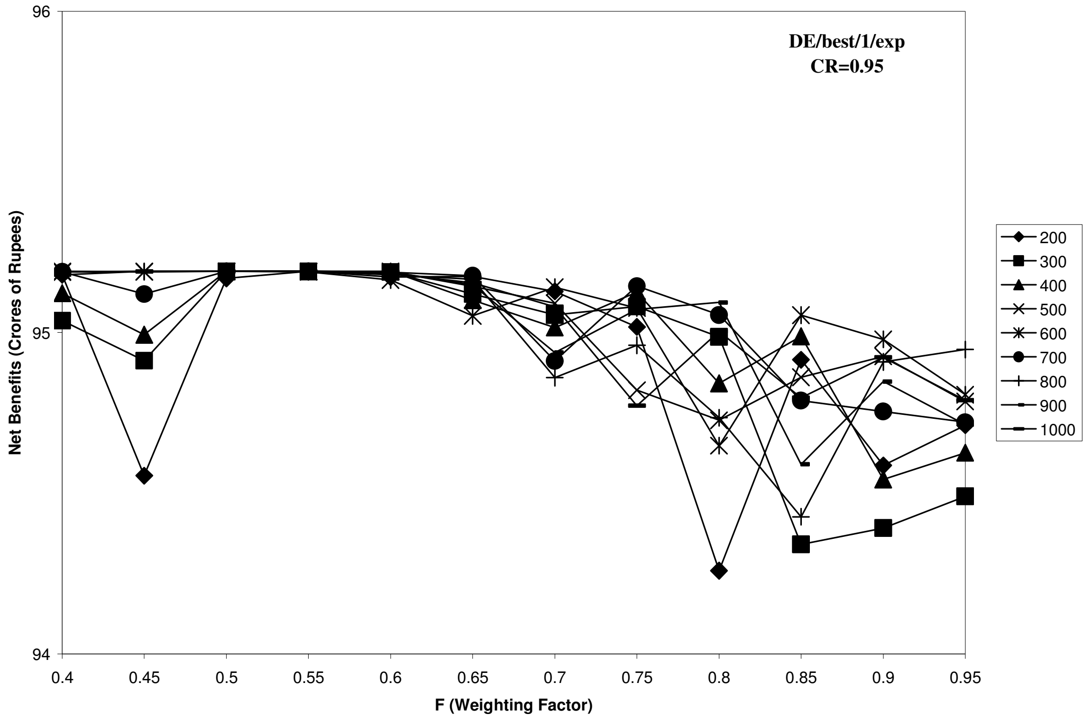

Fig. 6. Variation of weighting factor F for different NP values keeping CR =0.95 for strategy DE/best/1/exp

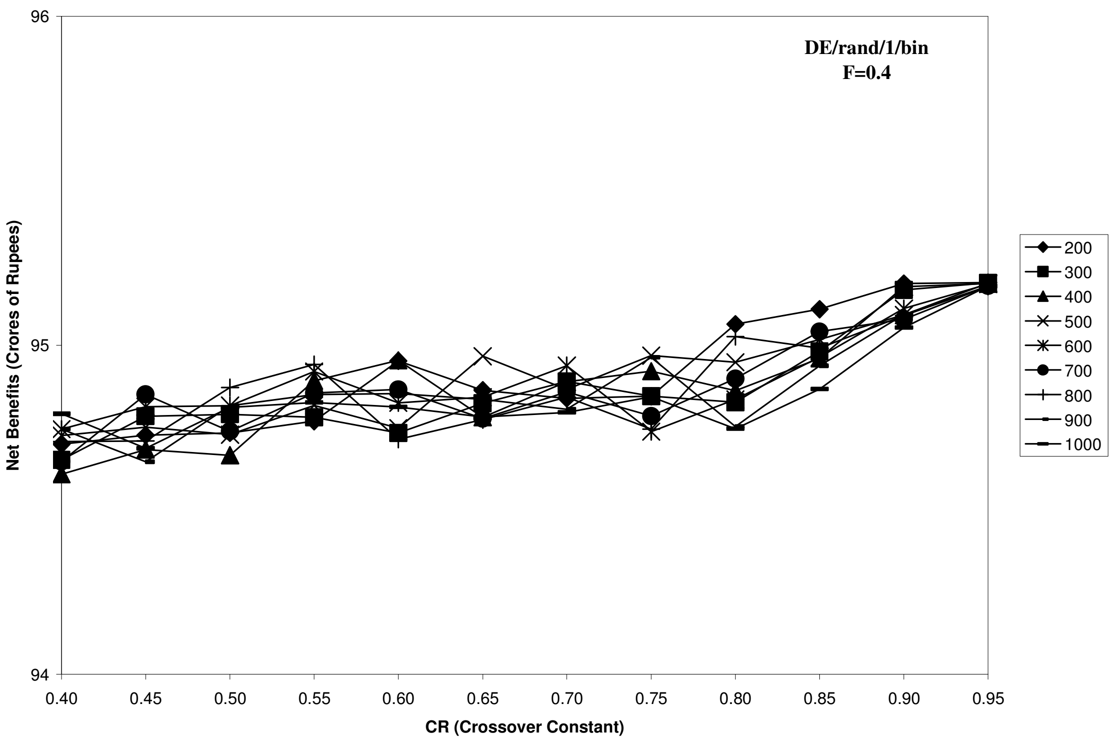

Fig. 7. Variation of crossover constant CR for different NP values keeping F=0.4 for strategy DE/rand/1/bin

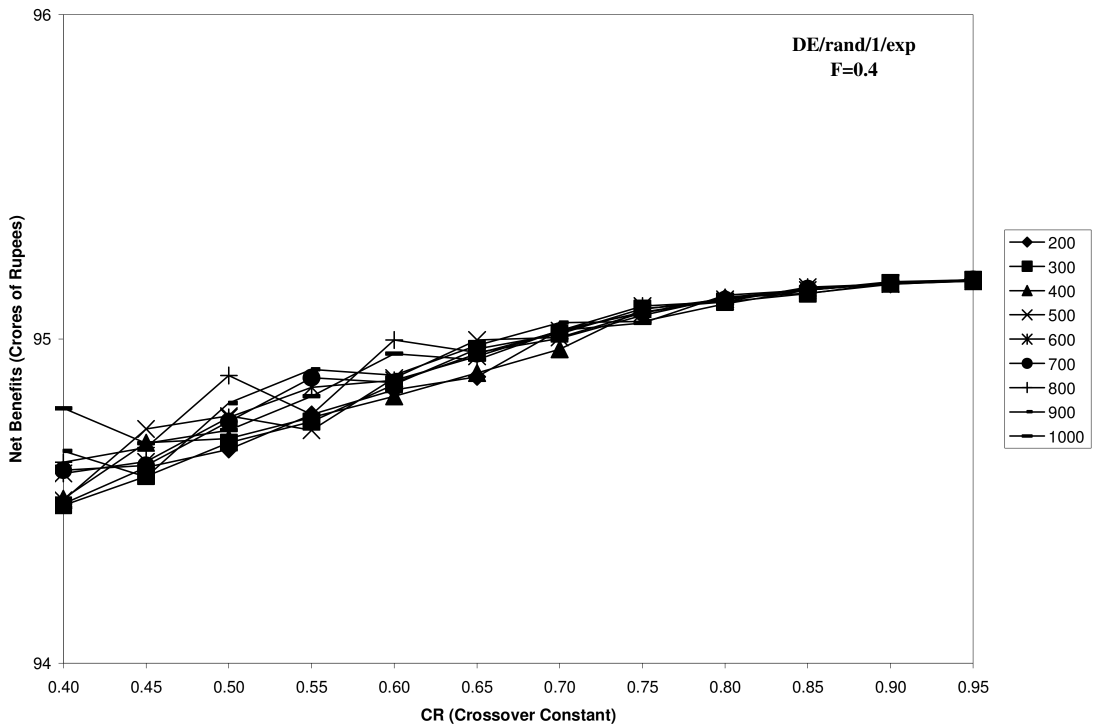

Fig. 8. Variation of crossover constant CR for different NP values keeping F=0.4 for strategy DE/rand/1/exp

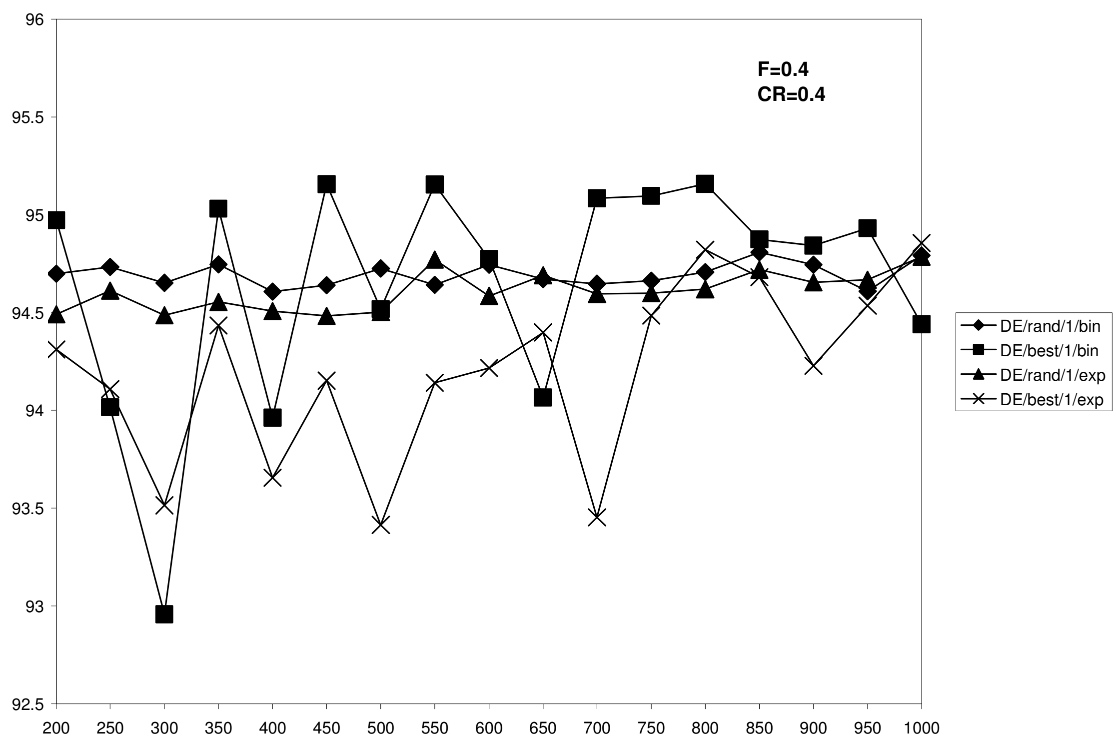

Fig 9. Variation of Population size NP for different strategies keeping F=0.4 and CR=0.4

CONCLUSIONS

DE is successfully applied to the irrigation planning problem of Bisalpur Project, Rajasthan, India. Effect of Population size (NP), Crossover constant (CR), Weighting factor (F) and ten different strategies (variations) of DE on benefits are studied. Results of DE are also compared with solution of Linear Programming (LP). It is observed that the net benefits (crores of rupees) obtained from ten DE strategies are 95.1901, 97.722 (constraint violation), 96.027 (constraint violation), 94.997 (constraint violation), 95.1903, 95.183, 95.1903, 95.1901, 95.1211 and 95.1903 whereas it is 95.1903 in case of LP. It is concluded that DE/rand-to-best/1/bin is the best strategy for the planning problem with maximum net benefits of 95.1903 crores of rupees taking minimum CPU time of 2.844 seconds as compared to LP. The present study can be extended to similar situations with suitable modifications.

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