**Progressive Multigrid Eigensolvers for Multiscale Spectral Segmentation**

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Overview

System Comparison

Multiscale

Progressive Multigrid Multiscale

Multilevel Eigensolvers with Applications to Segmentation of Parts and Pixels.

\[ \text{Eigenvector Convergence Comparison} \]

\[ \text{Multiscale Spectral Pb} \]

\[ \text{Runtime} \]

Let \((C, M)\) define a constrained Angular Embedding (AE) problem by specifying relationships between graph nodes: 

\[ v \rightarrow v \rightarrow \text{pairwise relative ordering matrix} \]

\[ C \rightarrow C \rightarrow \text{pairwise confidence matrix} \]

\[ U \rightarrow U \rightarrow \text{matrix of linear constraints} \]

\[ M \rightarrow M \rightarrow \text{matrix of non-lin. constraints} \]

Multiscale:

- Upgrade \(C, U\) to arrays \(C_i, U_i\) indexed by level \(s\).
- Pairwise relationships constrained to be within-level:

\[ v_s \rightarrow v_s \rightarrow \text{pairwise relative ordering matrix} \]

\[ C_s \rightarrow C_s \rightarrow \text{pairwise confidence matrix} \]

\[ U_s \rightarrow U_s \rightarrow \text{matrix of linear constraints} \]

\[ M_s \rightarrow M_s \rightarrow \text{matrix of non-lin. constraints} \]

Multigrid:

- Relax to generalized eigenproblem

\[ \text{Minimize:} \quad \sum_{l=1}^{L} \| \mathbf{Q}_l \mathbf{z}_l - \mathbf{P}_l \mathbf{z}_l \|^2 \quad \text{subject to:} \quad \mathbf{z}_l \in \mathcal{C}_l \]

- Multilevel Eigensolvers with Applications to Segmentation of Parts and Pixels.

\[ \text{Eigenvector Convergence Comparison} \]

\[ \text{Multiscale Spectral Pb} \]

\[ \text{Runtime} \]

One active problem instance in 0.001 sec. Baseline: \(6000 \times 4000\) pixels, 8 levels, 10000 eigenvalues.

Reference:


In this paper, we present a new approach for solving large-scale eigenspectrum problems arising from multiscale spectral segmentation. Our method, Progressive Multigrid Eigensolvers, leverages the hierarchical nature of the problem to significantly reduce computational cost. We demonstrate its effectiveness on a variety of real-world datasets, showing substantial speedup compared to traditional methods while maintaining comparable segmentation accuracy.