Angular Embedding:
from Jarring Intensity Differences to Perceived Luminance

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Distinction: Intensity, Brightness, and Lightness

intensity  = measured luminance:  \( I_1 > I_2 = I_3 > I_4 = I_5 > I_6 \)
brightness = perceived luminance:  \( B_1 > B_2 > B_3 > B_4 > B_6 > B_5 \)
lightness  = perceived reflectance:  \( L_1 = L_2 > L_3 = L_4 = L_6 > L_5 \)
Helmholtz and Hering Debate

1. Helmholtz: byproduct of high-level cognitive cause
   - recover reflectance from luminance with unknown illumination
   - Land & McCann, Retinex, 1971
   - Barrow & Tenenbaum, intrinsic images, 1978

2. in-between
   - Ross & Pessoa, selective integration model, 2000
   - Kelly & Grossberg, Form-And-Color-And-DEpth, 2000

3. Hering: manifestation of low-level physiological cause
   - lateral inhibition, center-surround filtering
   - Blakeslee et al, multiscale filtering, 2005
Basic Brightness Illusions

Simultaneous Contrast

White  Anti-snake  Snake
Textbook Explanation: Center-Surround Filtering

- center-surround filter = difference of Gaussians
Selective Enhancement is a Must but not by Size

increment-decrement

double-decrement

Enhancing small edges only explains one of the two illusions!
Insight: Selective Enhancement by Edge Geometry

Coarse-scale differences provide the right selective enhancement. Brightness differences across an edge increase with its curvature.
Brightness is Analogous to Motion Perception

1. **Feature** → *enable* brightness with short-range cues
   fine-scale for interiors, and coarser-scale across edges

2. **Aperture** → *reinforce* brightness with long-range cues
   paths of higher confidence, originating from corners, dominate

3. **Integration** → *realize* brightness from pairwise local cues
   maximally fulfill local orderings in accordance with confidence levels
Brightness Modeling is Global Brightness Ordering

1. edge detection
2. brightness ordering
3. angular embedding

intensity $I$
pairwise edges $E$
pairwise differences $(O, C)$
brightness $B$

$B - I$
New Integration Method: Angular Embedding

**input:** local ordering
- \( O = \) pairwise differences
- \( C = \) confidence in \( O \)

**output:** global ordering
- \( x = \) positions on a line, or
- \( z = \) positions on the unit circle

**old:** linear space
\[ x(a) - x(b) = O(a, b) \]

**new:** angular space
\[ \theta(a) - \theta(b) = O(a, b) \]
Criterion: Minimize Distance to Local Average

minimize: \[ \varepsilon(z; O, C) = \sum_a D(a, a) \cdot |z(a) - \tilde{z}(a)|^2 \]

local average: \[ \tilde{z}(a) = \sum_b \frac{C(a, b)}{D(a, a)} z(b) e^{iO(a, b)} \]

total confidence: \[ D(a, a) = \sum_b C(a, b) \]
Optimum: Angles of the Smallest Eigenvector

**Angular Embedding**

minimize: \( \varepsilon(z; O, C) = z' W z \)

representation: \( z = e^{i \theta} \)

error: \( W = (I - D^{-1} M)' D (I - D^{-1} M) \)

measurement: \( M = C \cdot e^{iO} \)

degree: \( D = \text{Diag}(C1) \)

optimum: \( \theta^* = \angle z^* = \angle \text{smallest-eigenvector-of } (W, D) \)

**Least Squares**

minimize: \( \varepsilon(x; O, C) = \sum C(a, b)(x(a) - x(b) - O(a, b))^2 \)

measurement: \( M = C \cdot O + (C \cdot O)' \)

degree: \( D = \text{Diag}((C + C')1) \)

transition: \( P = D^{-1}(C + C') \)

optimum: \( x^* = (I - P)^{-1} \cdot (D^{-1} M1) \)
An Efficient and More Robust Integration Method

original image

3 × 6 measurement outliers
neighbourhood radius = 2

LS optimum $x^*$

AE optimum $z^*$

AE optimum $\theta^*$
Brightness as Intensity Deviating along Gradient

Adelson, 1999: X junctions & atmospheres

deviation by scene interpretation
deviation by intensity context itself

$$\frac{14}{15}$$
Brightness as Gestalt from Scale-Mixed Differences

input: objective intensity

output: subjective brightness

brightness – intensity

Simultaneous Contrast  Anti-Snake  Snake  Koffka Ring  Benary Cross