Understanding Popout through Repulsion

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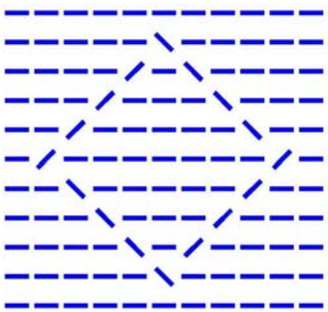
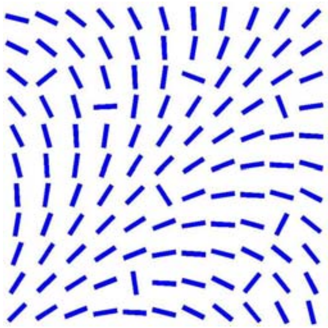
Questions to Be Asked

- What is popout?



finding patterns



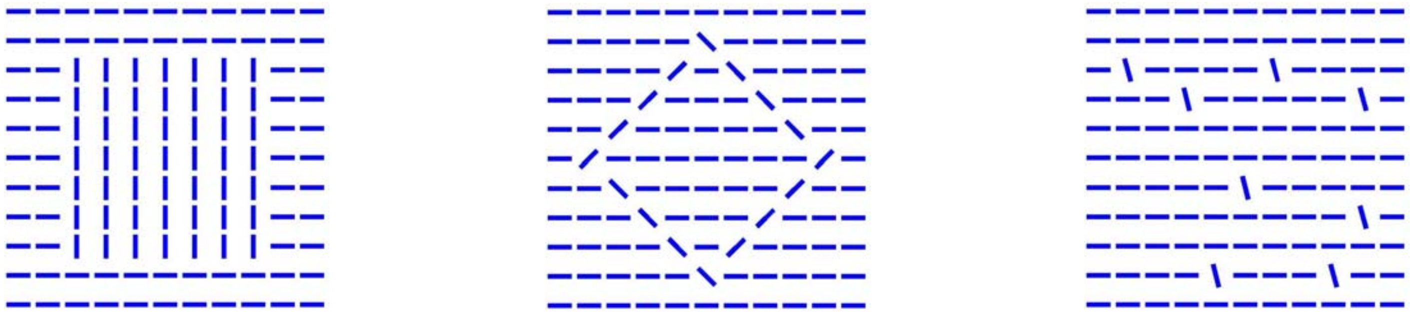
finding outliers

- When can popout be perceived?

  What grouping factors are needed to bring about popout?

  What grouping criteria can capture most popout phenomena?

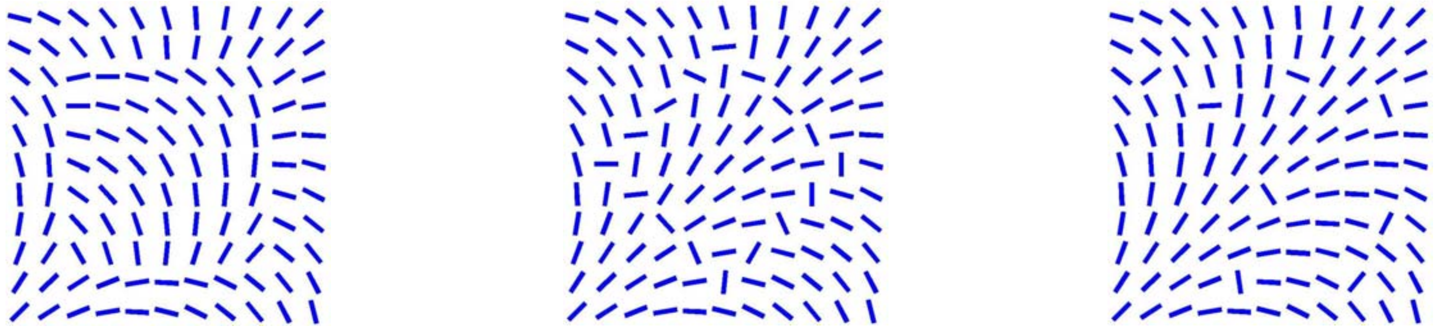
Observation: Popout by Feature Similarity



- Similarity grouping assumes that groups are characterized by unique features which are homogeneous across members. Segmentation is then a feature discrimination problem between different regions.

- Feature discrimination only works when the similarity of features within areas confounds with the dissimilarity between areas, illustrated in the above examples of region segmentation, contour grouping and popout.

Observation: Popout by Feature Contrast



- When feature similarity within a group and feature dissimilarity between groups are teased apart, the two aspects of grouping, association and segregation, can contribute independently to perceptual organization.

- In particular, local feature contrast plays an active role in binding even dissimilar elements together.

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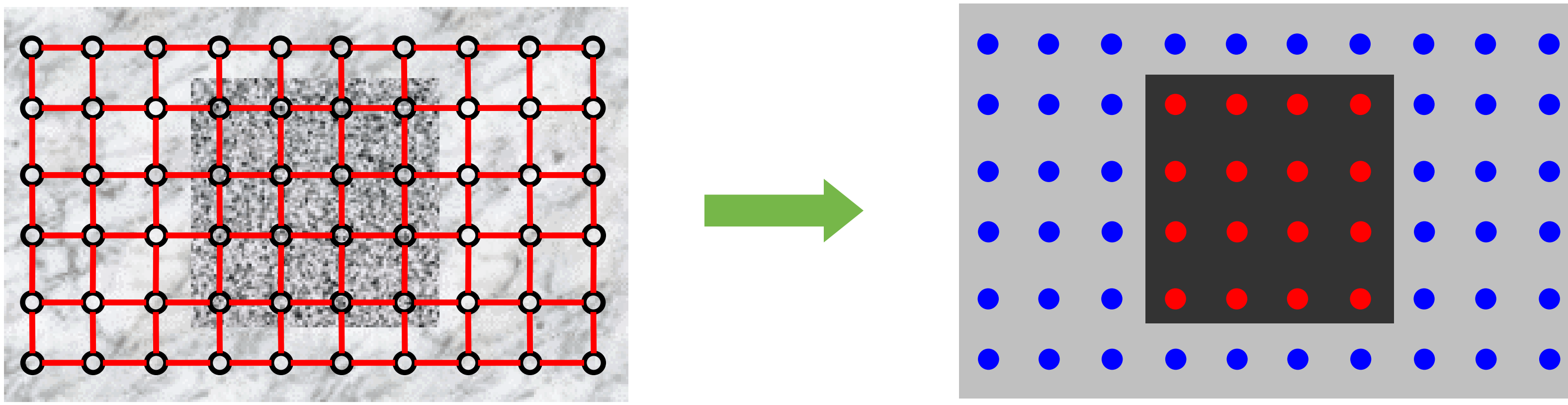
Model: Popout as Contextual Grouping

- Attraction measures the degree of feature similarity. It is used to associate members within groups.
- Repulsion measures the degree of feature dissimilarity. It is used to segregate members belonging to different groups.
- Contextual grouping consists of dual procedures of association and segregation, with coherence detection and salience detection at the two extremes of the spectrum.

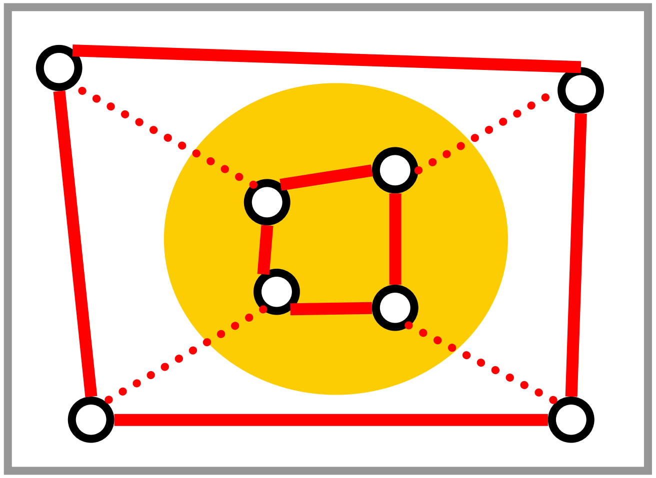
Representation: Relational Graphs



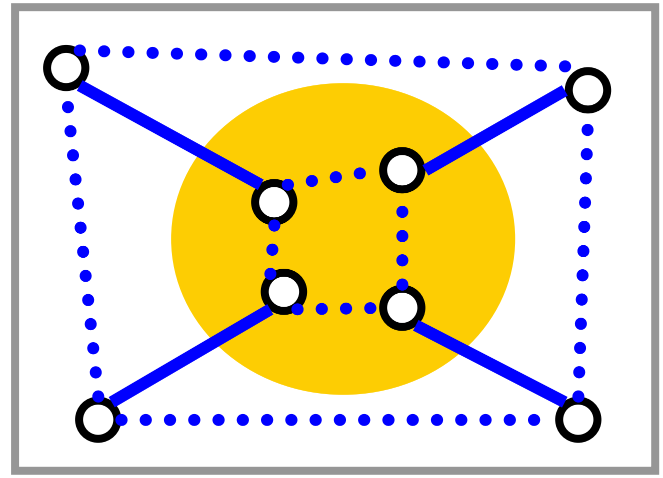
- G=(V, E, A, R)
  - V: each node denotes a pixel
  - E: each edge denotes a pixel-pixel relationship
  - A: each weight measures pairwise similarity
  - R: each weight measures pairwise dissimilarity

- Segmentation = node partitioning
  - break V into disjoint sets V_1 , V_2

Criteria: Dual Goals on Dual Measures



Cut-off attraction is the separation cost



Cut-off repulsion is the separation gain

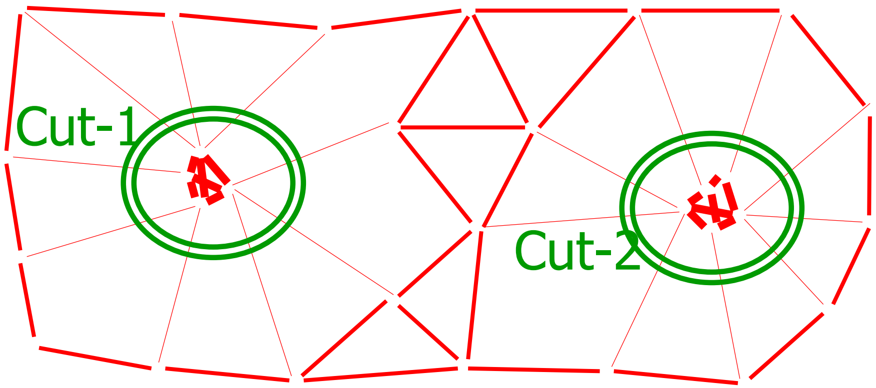
- Maximize

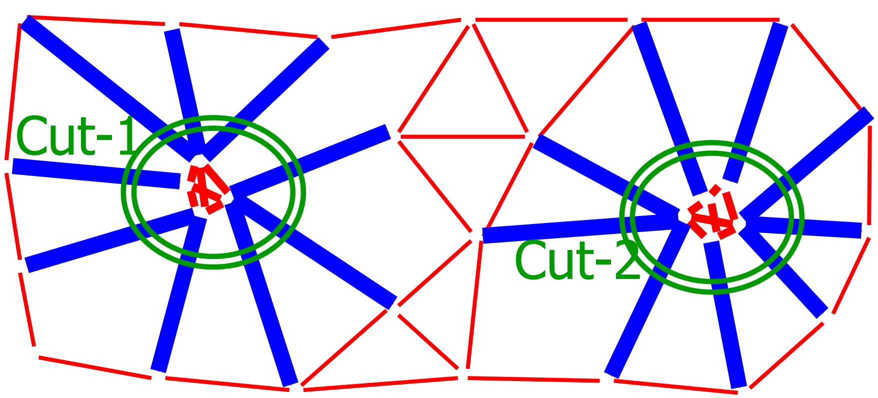
  within-region attraction and between-region repulsion

- Minimize

  between-region attraction and within-region repulsion

Criteria: Why Repulsion Help Popout?





- Cost-1 + Cost-2 > min (Cost-1, Cost-2)
- Attraction unites elements who have **<u>common friends</u>**
- Cost-1 – Gain-1 + Cost-2 – Gain-2 < min (Cost-1 – Gain-1 + Gain-2, Cost-2 – Gain-2 + Gain-1)
- Repulsion unites elements who have **<u>common enemies</u>**

Model: Energy Function Formulation

$$X_l(u) = \begin{cases} 1, & u \in V_l \\ 0, & u \notin V_l \end{cases}$$

- Group indicators

$$W = A - R + D_R$$

- Weight matrix

$$D = D_A + D_R$$

- Degree matrix

$$y = (1-\alpha)X_1 - \alpha X_2, \quad \alpha = \frac{\deg(V_1)}{\deg(V)}$$

- Change of variables

$$Nassoc(X_1, X_2) = \sum_{t=1}^{2} \frac{X_t^T W X_t}{X_t^T D X_t} = \frac{y^T W y}{y^T D y}$$

- Energy function as a Rayleigh quotient

$$\max \frac{y^T W y}{y^T D y} \quad \Rightarrow \quad W\ y = \lambda_1\ D\ y$$

- Eigenvector as solution

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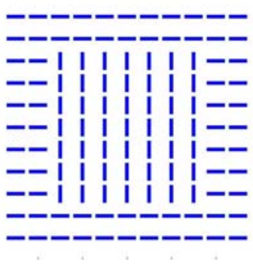
Interpretation: Eigenvector as a Solution

- The derivation holds so long as $X_1 + X_2 = 1$

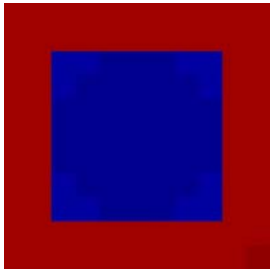
$$y = (1-\alpha)X_1 - \alpha X_2 = X_1 - \alpha$$

- The eigenvector solution is a linear transformation, scaled and offset version of the probabilistic membership indicator for one group.

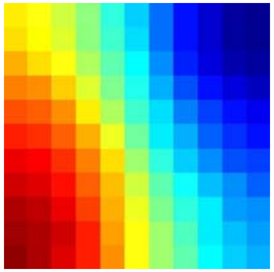
- If y is well separated, then two groups are well defined; otherwise, the separation is ambiguous



stimulus



Solution y
well separated



Solution y
ambiguous

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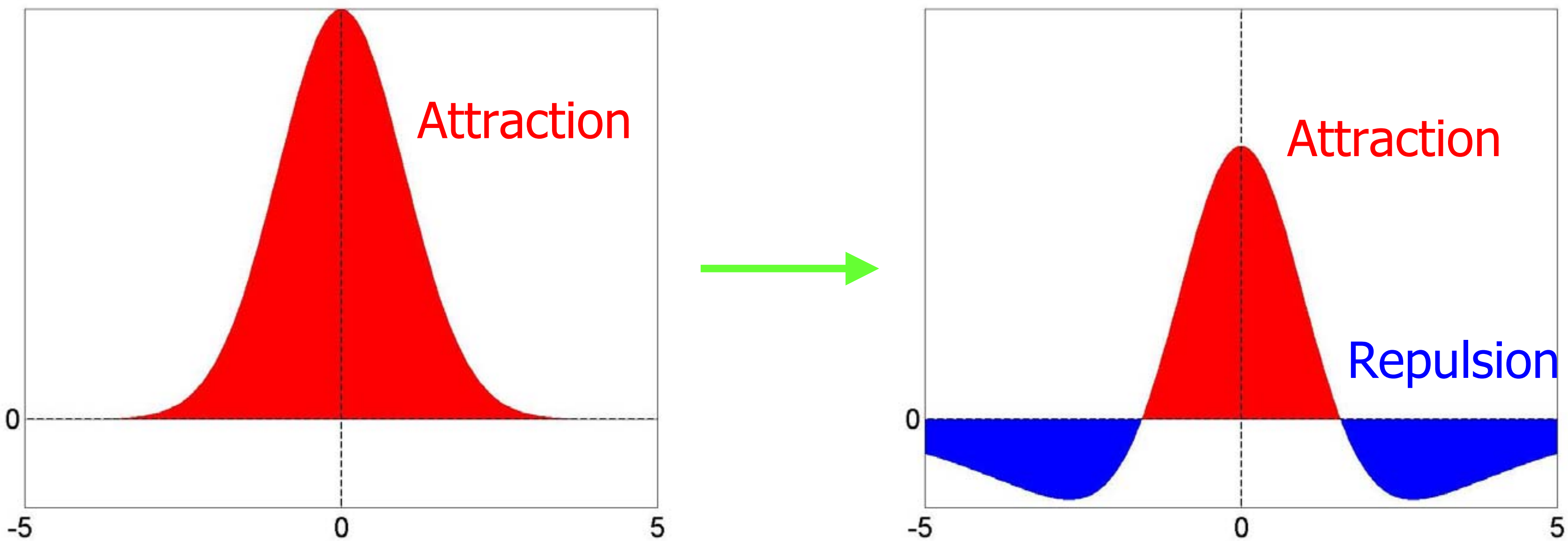
Interaction: from Gaussian to Mexican Hat

$$W = A - R + D_R$$



$$W_{ij} = e^{-\frac{(f_i - f_j)^2}{2\sigma_1^2}} - \frac{\sigma_1}{\sigma_2} e^{-\frac{(f_i - f_j)^2}{2\sigma_1^2} \cdot \left(\frac{\sigma_1}{\sigma_2}\right)^2}$$

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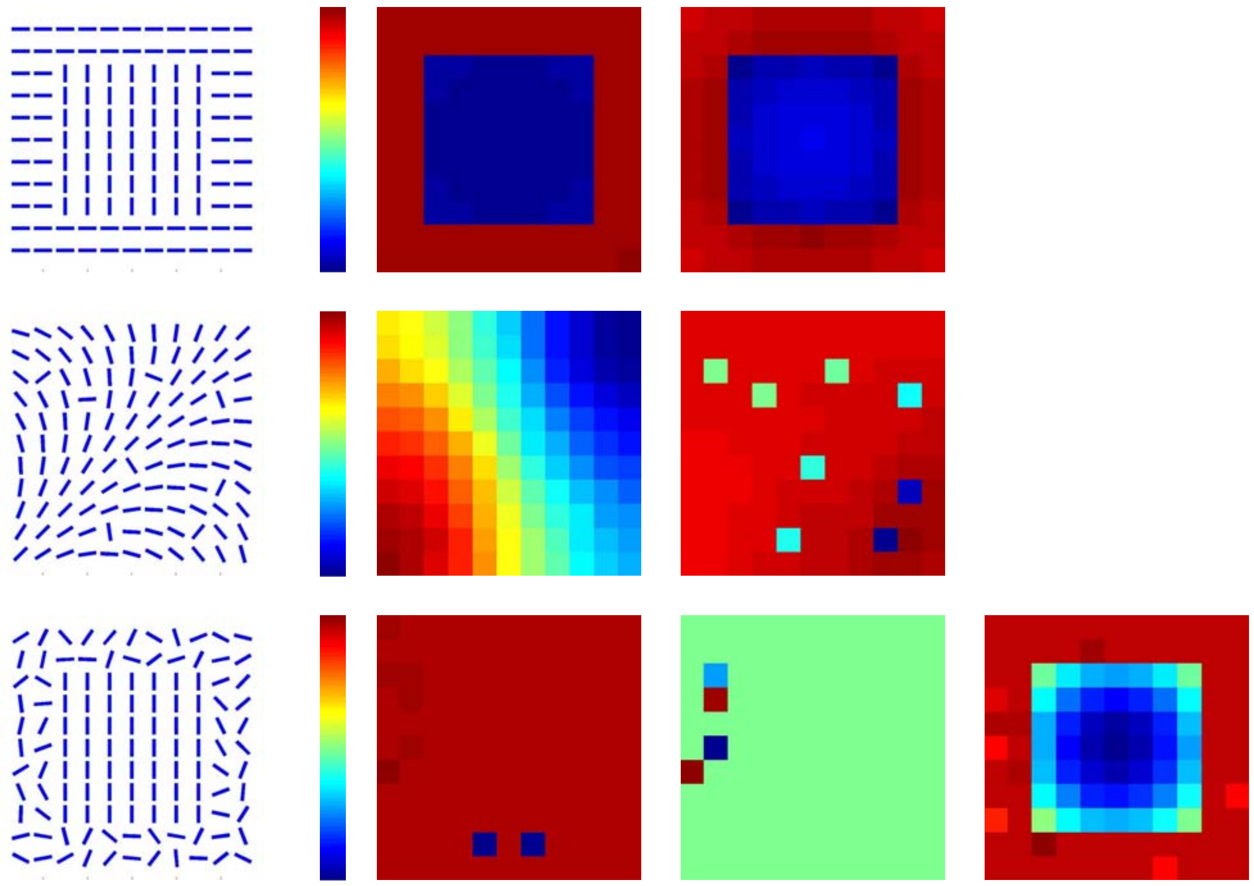
Regularization

$$W = \underbrace{W_+}_{A} - \underbrace{(-W_-)}_{R} \qquad\qquad W = \underbrace{(W_+ + \Delta)}_{A} - \underbrace{(\Delta - W_-)}_{R}$$

$$(W + D_R, D) \qquad\qquad (W + D_R + D_\Delta, D + 2D_\Delta)$$

- Regularization does not depend on the particular form of Δ. Only D_Δ matters. To avoid bias, we choose $D_\Delta = \delta I$.
- Regularization equalizes two partitions by:
  - Decrease the relative importance of large attraction
  - Decrease the relative importance of large repulsion

Results: Popout



Stimuli Attraction +Repulsion +Regularization

- ➔ Attraction to bind similar elements
- ➔ Repulsion to bind dissimilar elements
- ➔ Regularization to equalize

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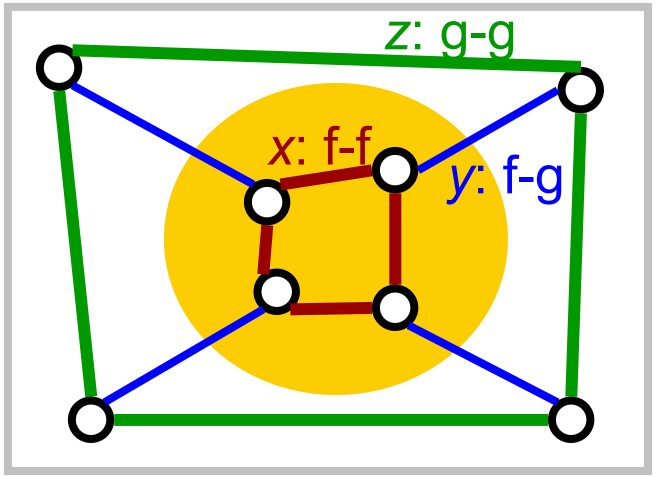
When Can a Figure Popout



- *x* : figure-figure connection
  y : figure-ground connection
  z : ground-ground connection
- Attraction: x , y , z >0
  Repulsion: x , y , z <0
- Coherent: attraction within a group
  Incoherent: repulsion within a group

- Question 1:
  What are the feasible sets of (x,y,z)
  so that figure-ground can be separated as is ?

- Question 2:
  How do the feasible sets change with the degree of
  regularization $D_\Delta = \delta I$?

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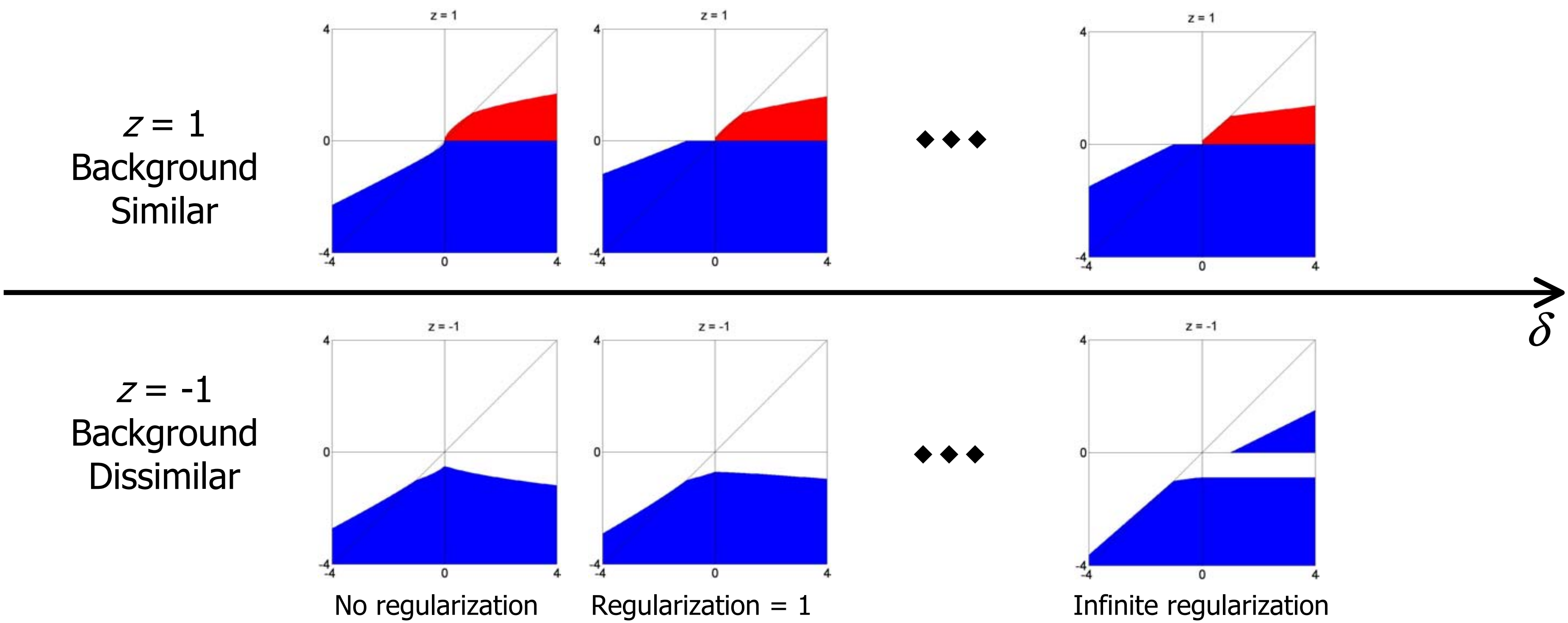
Conditions for Popout: Normalized Cuts

		No regularization		Infinite regularization	
		$\delta = 0$		$\delta = \infty$	
Background Similar	$z = 1$	$x \in \left(1 - y - \sqrt{1 - 2y + 9y^2}, +\infty\right),$	$y \in (-\infty, 0)$	$x \in (-1 + 2y, +\infty),$	$y \in (-\infty, 0)$
		$x \in \left(\frac{2y^2}{1+y}, +\infty\right),$	$y \in [0, 1]$	$x \in \left(\max\left(0, \frac{-1+8y}{7}\right), +\infty\right),$	$y \in [0, 1]$
		$x \in \left(-y + 2y^2, +\infty\right),$	$y \in (1, +\infty)$	$x \in (-7 + 8y, +\infty),$	$y \in (1, +\infty)$
Background Dissimilar	$z = -1$	$x \in \left(\frac{-2y^2}{1-y}, \frac{-1+2y+8y^2}{2}\right),$	$y \in (-\infty, -1)$	$x \in \left(\frac{1+8y}{7}, +\infty\right),$	$y \in (-\infty, -1)$
		$x \in \left(-y - 2y^2, \frac{-1+2y+8y^2}{2}\right),$	$y \in \left[-1, -\frac{1}{2}\right]$	$x \in (7 + 8y, +\infty),$	$y \in \left[-1, -\frac{7}{8}\right]$
		$x \in \phi,$	$y \in \left(-\frac{1}{2}, +\infty\right)$	$x \in \phi,$	$y \in \left(-\frac{7}{8}, 0\right)$
				$x \in (1 + 2y, +\infty),$	$y \in [0, +\infty)$

- Scale on *x,y,z* does not change the grouping results
- Linear or quadratic bounds on x-y

Conditions for Popout: Normalized Cuts



- Repulsion(blue) greatly expands feasible regions.
- Regularization helps esp. when within-group connections are weak.

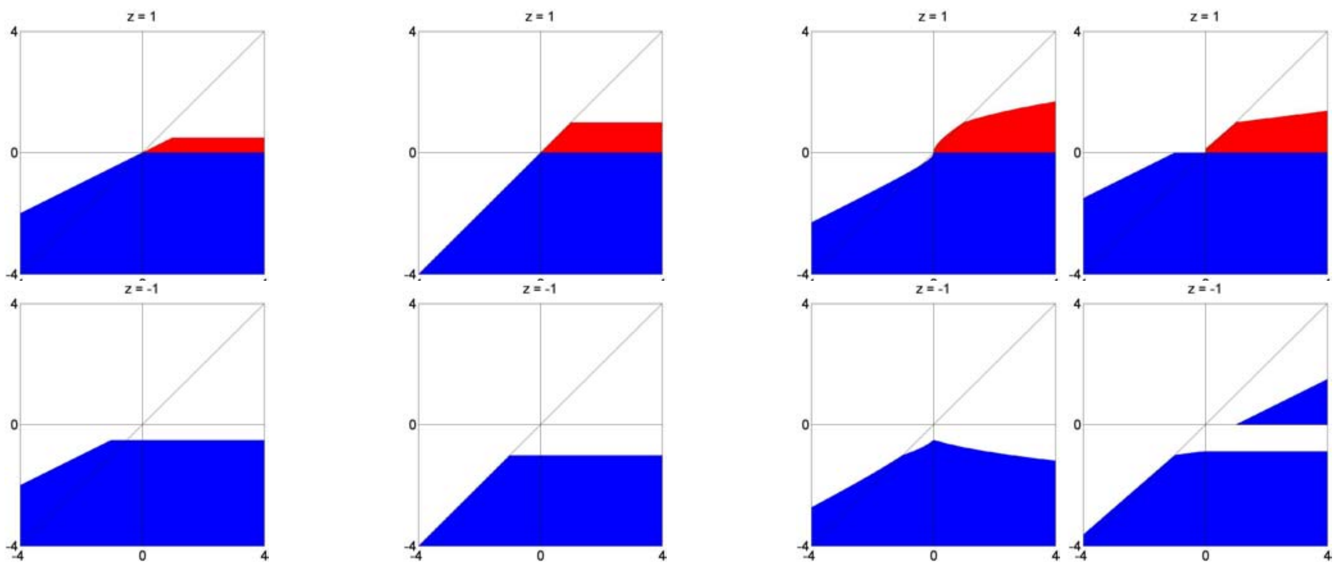
Comparison of Grouping Criteria

	Min Cuts	Average Cuts
$z=1$	$x \in (2y, \infty),\quad y \in (-\infty, 0.5)$	$x \in (y, \infty),\quad y \in (-\infty, 1)$
$z=-1$	$x \in (2y, \infty),\quad y \in (-\infty, -0.5)$	$x \in (y, \infty),\quad y \in (-\infty, -1)$

- ➔ Repulsion helps as well
- ➔ Invariance to regularization
- ➔ Linear bounds on x-y
- ➔ Narrower than Normalized Cuts

z = 1
Background
Similar

z = -1
Background
Dissimilar



No regularization Infinite regularization

Min Cuts Average Cuts Normalized Cuts

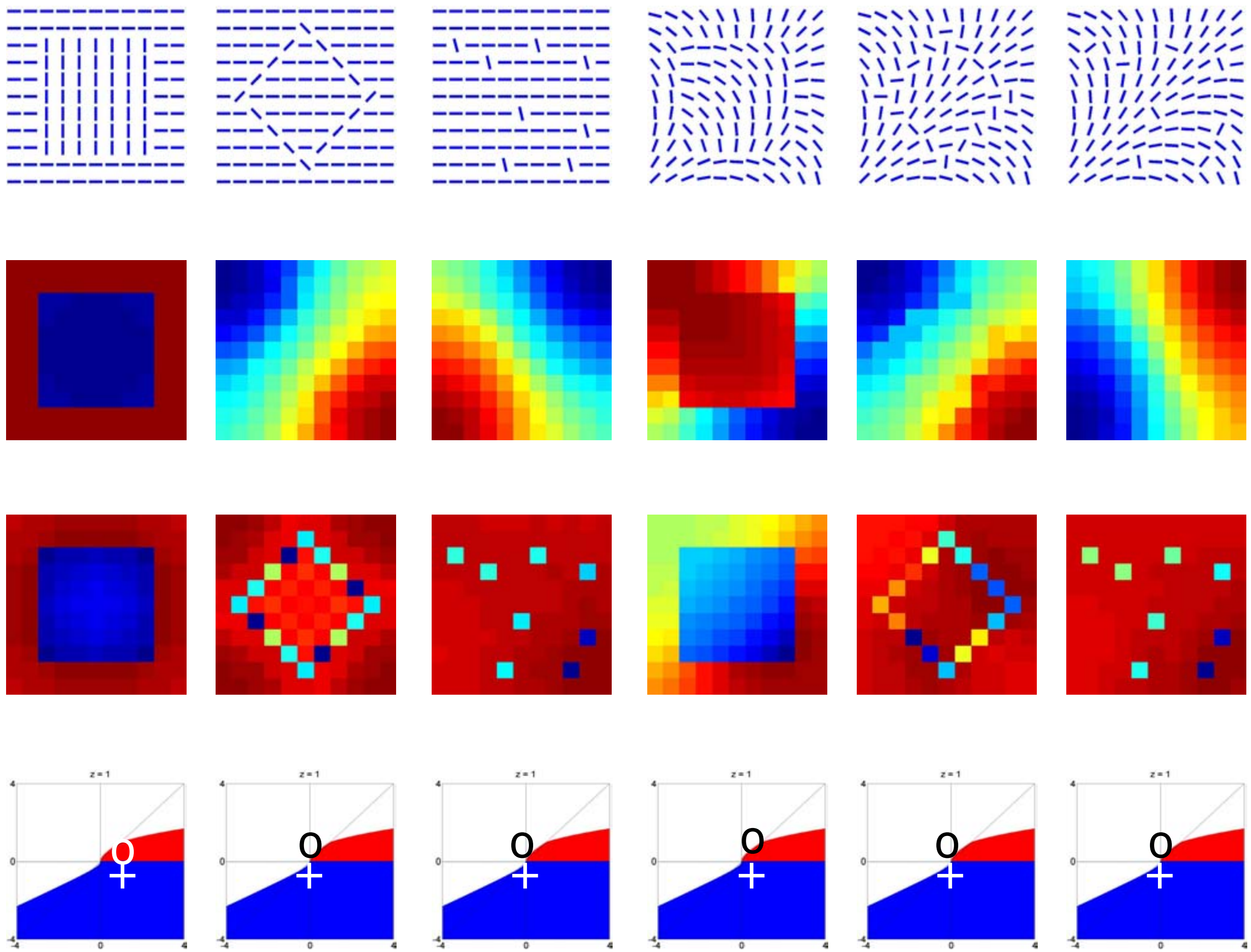
Results: Popout in Coherent Backgrounds

Stimuli

Solution w/ Attraction

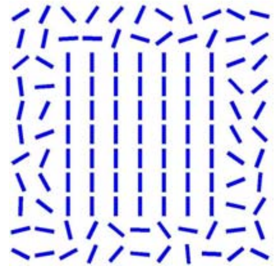
Solution w/ Repulsion

marked on Feasibility map
"o": Attraction
"+": Repulsion



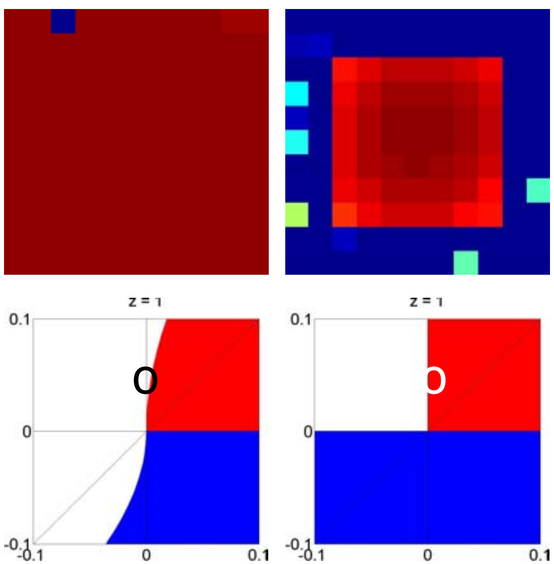
Results: Popout in Random Backgrounds

Solution with
Regularized
Attraction



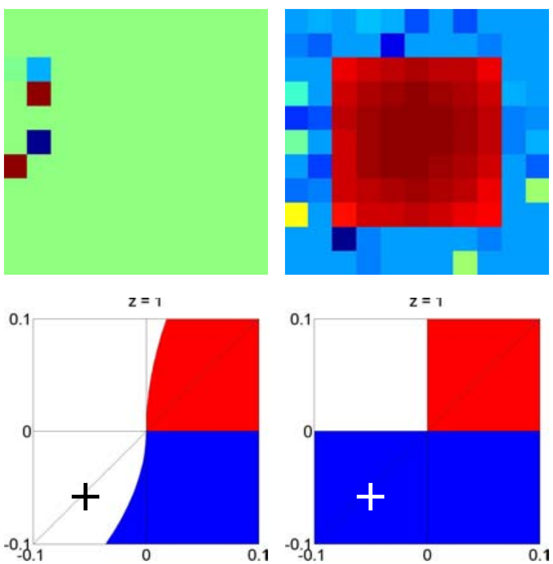
stimulus

Solution with
Regularized
Repulsion



δ = 0 per w δ = 0.05 per w

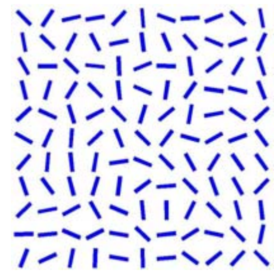
- No grouping w/o regularization.
- Repulsion helps as well.
- Insensitive to the degree of regularization.



δ = 0 per w δ = 0.05 per w

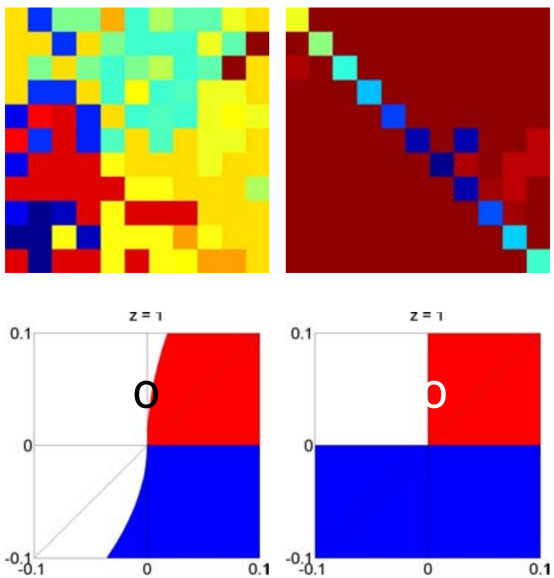
Results: Popout in Random Backgrounds

Solution with Regularized Attraction



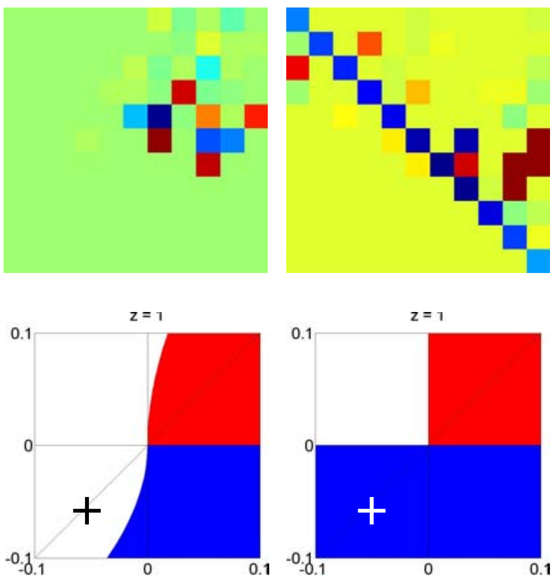
stimulus

Solution with Regularized Repulsion



δ = 0 per w δ = 0.05 per w

- ➔ No grouping w/o regularization.
- ➔ Repulsion helps as well.
- ➔ Insensitive to the degree of regularization.



δ = 0 per w δ = 0.05 per w

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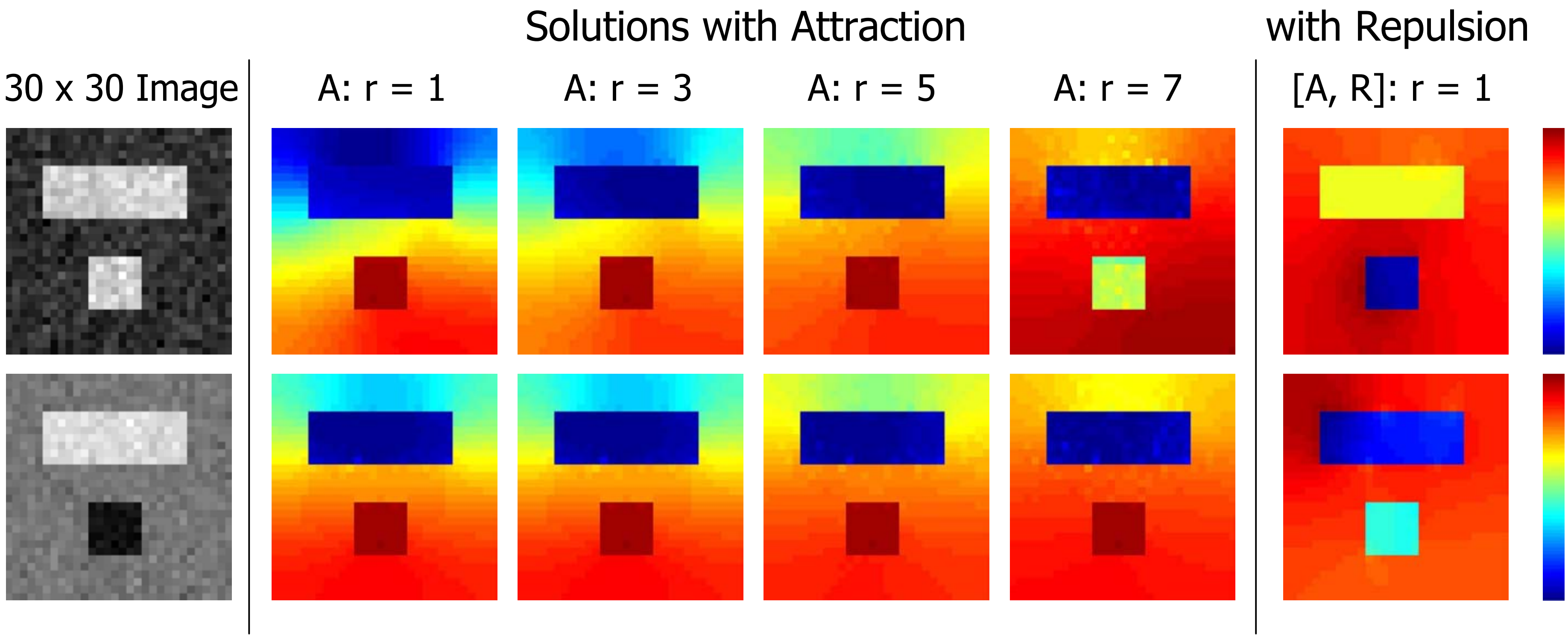
Results: Computational Efficiency



- ➔ If only attraction is allowed, much larger neighbourhood radius is needed to bring similar subregions together.
- ➔ When subregions are dissimilar, increasing radius does not help attraction to bring them together.

Conclusions

- Pairwise relationships
  - ➔ Attraction: similarity grouping
  - ➔ Repulsion: dissimilarity grouping
- Advantages of repulsion
  - ➔ Complementary: regularization
  - ➔ Computational efficiency
- Figure-ground organization

	Coherent ground	Incoherent ground
Coherent figure	Attraction	+Regularization
Incoherent figure	+Repulsion	

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