Experimental Design for Machine Learning on Multimedia Data
Lecture 3

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Discuss Homework

- Homework 1: Any questions
- Homework 2: Let’s do it.
- Homework 3: online today and also in paper here

Please start forming teams.

Rishi is now official and has office hours to discuss homework & project.

Reminder: The New Scientific Method

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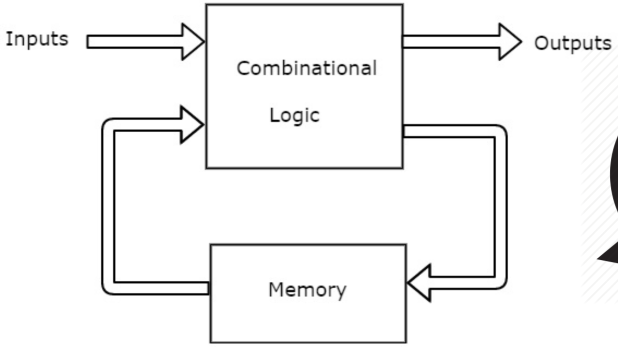
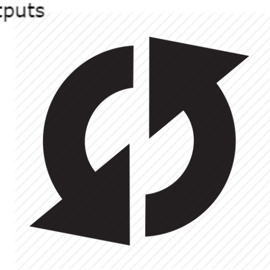




$$E = mc^2$$

Reminder: Thought Framework Machine Learning

- Intelligence: *The ability to adapt* (Binet and Simon, 1904)

- Machine learning *adapts a finite state machine M to an unknown function based on observations*.

- Input: *n* rows of observations (instances) in a table with header:

$$(x_1, x_2, \ldots, x_m, f(\overrightarrow{x}))$$

  where $f(\overrightarrow{x})$ is a column with labels we call target function.

- Output: State machine *M* that maps a point

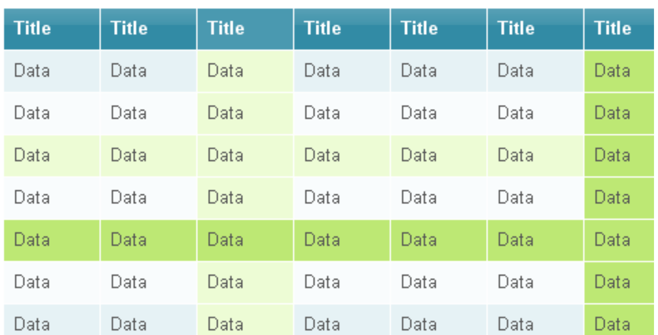
$$(x_1, x_2, \ldots, x_m) \implies f(\overrightarrow{x})$$

Thought Framework: Machine Learning

- Assume

$$x_i \in \mathbb{R}, f(\overrightarrow{x}) \in \{0,1\}$$

(binary classifier)



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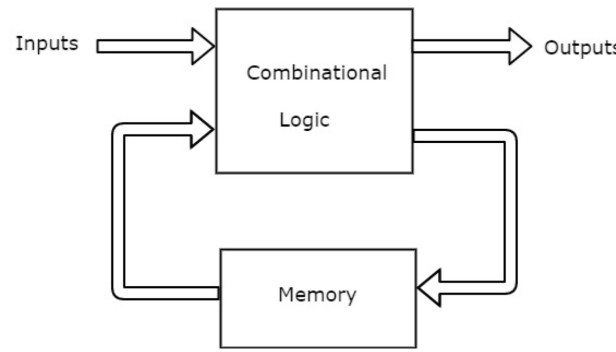


- Question:

How many state transitions does *M* need to model the training data?

Refresh: Memory Arithmetic

- *Information is reduction of uncertainty*:
  $H = -\log_2 P = -\log_2 \frac{1}{\#states} = \log_2 \#states$
  measured in bits.

- Information: $\log_2 \#states$ (positive bits)
  Uncertainty: $\log_2 P = \log_2 \frac{1}{\#states}$ (negative bits)

- If states are not equiprobable, *Shannon Entropy* provides tighter bound.

  Important for homework!

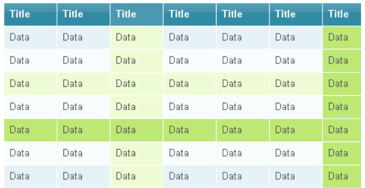
Thought Framework: Machine Learning

Assume

$$x_i \in \mathbb{R}, f(\overrightarrow{x}) \in \{0,1\}$$

(binary classifier)

Question:



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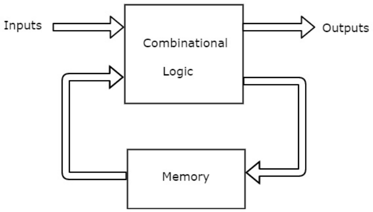

How many state transitions does *M* need to model the training data?

Maximally: #rows (lookup table)
Minimally: ?

Thought Framework: Machine Learning

- ***Intellectual Capacity:*** *The number of unique target functions a machine learner is able to represent (as a function of the number of model parameters).*

- ***Memory Equivalent Capacity (MEC):*** *A machine learner's intellectual capacity is memory-equivalent to N bits when the machine learner is able to represent all 2^N binary labeling functions of N uniformly random inputs.*

- At MEC or higher, *M* is able to **memorize** all possible state transitions from the input to the output.

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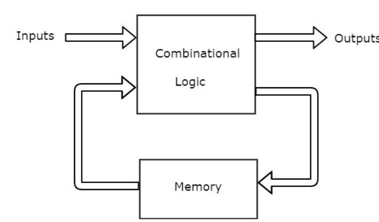


Project Question 2

- What is the estimated memory-equivalent capacity of the data you have for the machine learner you are using?
- Is there bias? If so: Normalize it.
- Test at least 3 different machine learners.

This Talk: Main trick

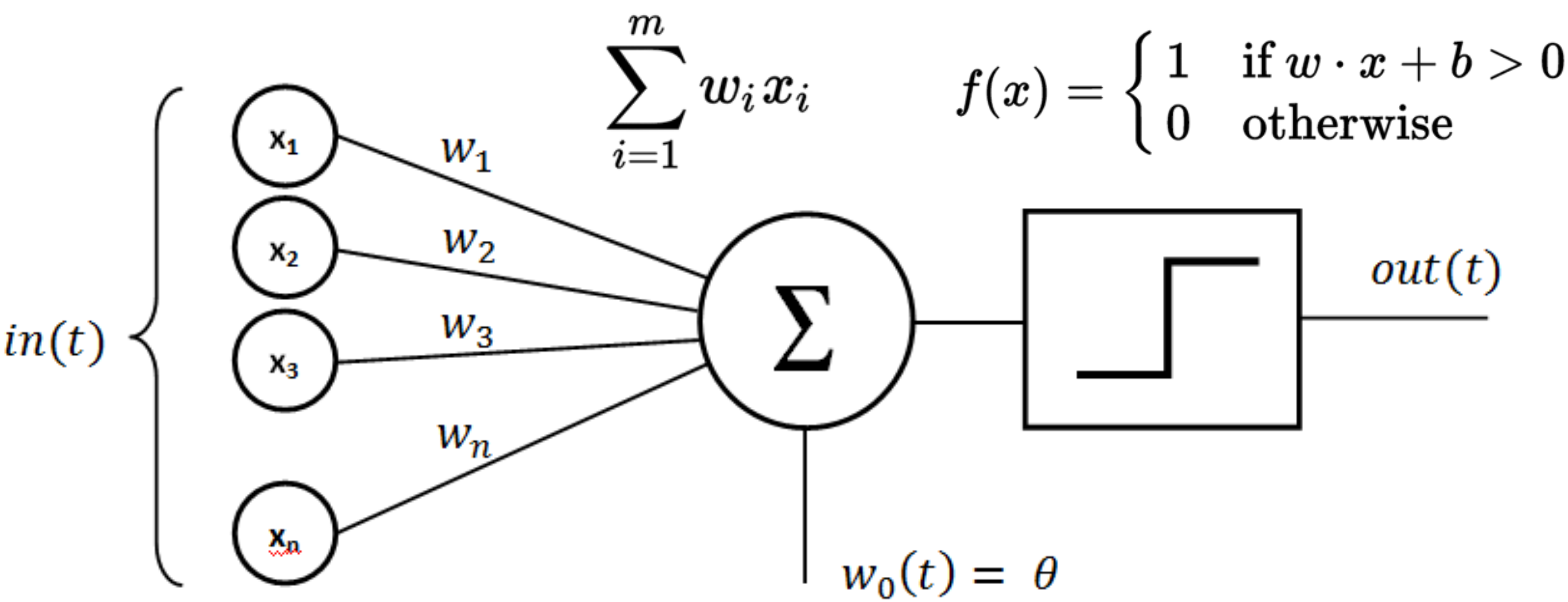
Memorization is worst-case generalization

- Using more parameters than needed for memorization is a waste of resources (CPU, memory, I/O, engineer tuning time).
- Using as many parameters as needed for memorization will most likely not generalize to a held-out data set. This, the machine learner overfits.
- Reducing parameters below memorization capacity will, in the best case, make the machine learner forget what's not relevant: generalization.

Machine Learning as Engineering Discipline

- Supervised **Machine Learners have a memory capacity in bits** that is **computable** and **measurable**.
    - Artificial Neural Networks with gating functions (Sigmoid, ReLU, etc.) have
        - a capacity upper limit that can be determined analytically using 4 principles
        - an effective capacity that can be measured on actual implementations.
- Predicting and measuring capacity allows for task-independent optimization of a concrete network architecture, learning algorithm, convergence tricks, etc…
- Capacity requirement can be approximately predicted given the input data and ground truth.
- Generalization can fail as a result of input redundancies. Occam's Razor helps to minimize the risk.

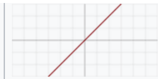
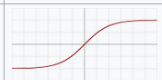
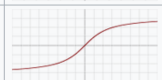
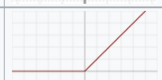
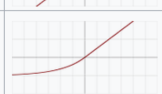
Repeat: The Perceptron



Physical interpretation: Energy threshold

Source: Wikipedia

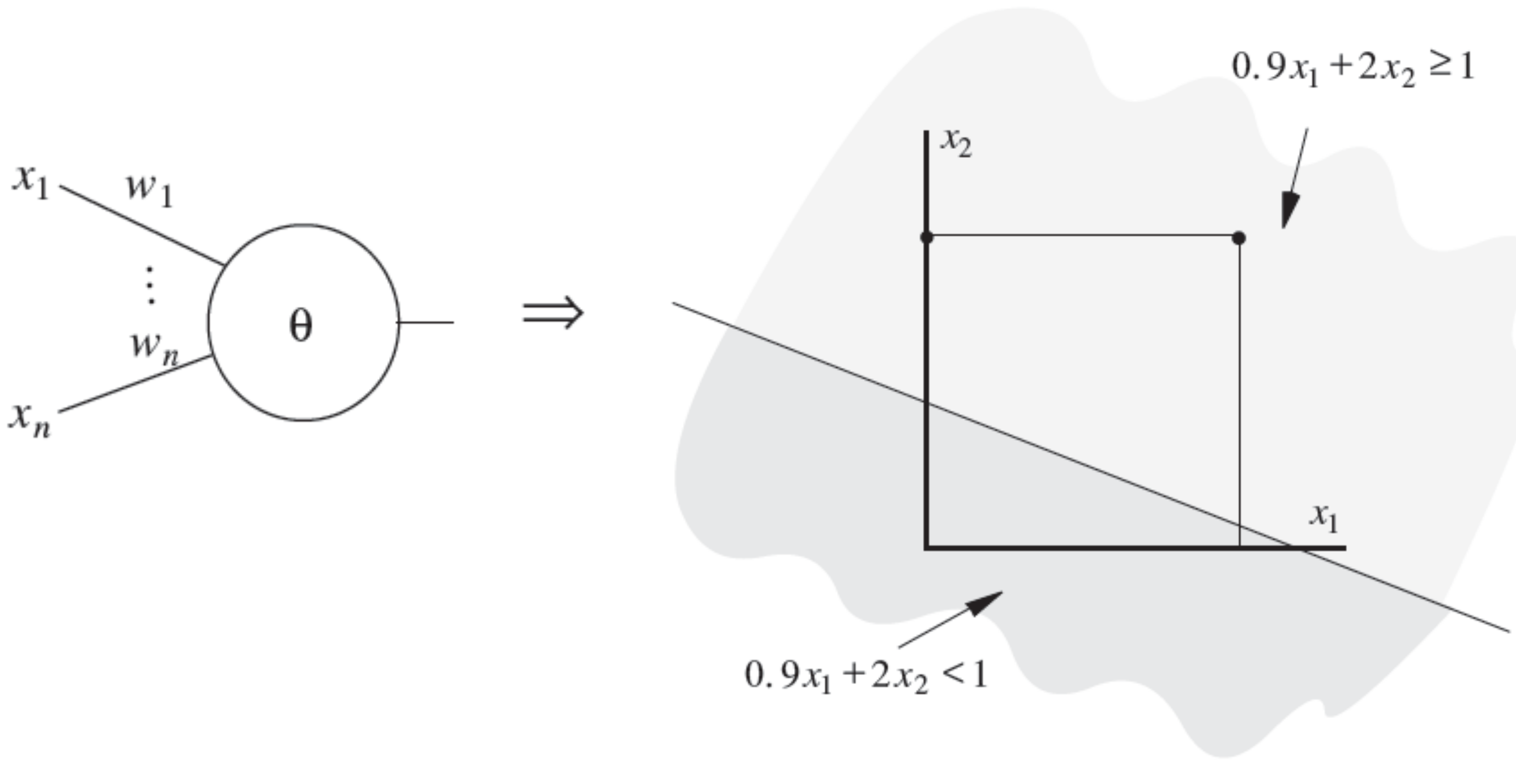
Repeat: Activation Functions (too many)

Identity		$f(x) = x$	$f'(x) = 1$	$(-\infty, \infty)$	C^∞
Binary step		$f(x) = \begin{cases} 0 & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for} & x \ne 0 \\ ? & \text{for} & x = 0 \end{cases}$	$\{0, 1\}$	C^{-1}
Logistic (a.k.a. Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$	$(0, 1)$	C^∞
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$	$(-1, 1)$	C^∞
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	C^∞
Softsign [7][8]		$f(x) = \frac{x}{1 + \lvert x \rvert}$	$f'(x) = \frac{1}{(1 + \lvert x \rvert)^2}$	$(-1, 1)$	C^1
Rectified linear unit (ReLU)[9]		$f(x) = \begin{cases} 0 & \text{for} & x < 0 \\ x & \text{for} & x \ge 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$[0, \infty)$	C^0
Leaky rectified linear unit (Leaky ReLU)[10]		$f(x) = \begin{cases} 0.01x & \text{for} & x < 0 \\ x & \text{for} & x \ge 0 \end{cases}$	$f'(x) = \begin{cases} 0.01 & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$(-\infty, \infty)$	C^0
Parameteric rectified linear unit (PReLU)[11]		$f(\alpha, x) = \begin{cases} \alpha x & \text{for} & x < 0 \\ x & \text{for} & x \ge 0 \end{cases}$	$f'(\alpha, x) = \begin{cases} \alpha & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$(-\infty, \infty)$	C^0
Randomized leaky rectified linear unit (RReLU)[12]		$f(\alpha, x) = \begin{cases} \alpha x & \text{for} & x < 0 \\ x & \text{for} & x \ge 0 \end{cases}$ [1]	$f'(\alpha, x) = \begin{cases} \alpha & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$(-\infty, \infty)$	C^0
Exponential linear unit (ELU)[13]		$f(\alpha, x) = \begin{cases} \alpha(e^x - 1) & \text{for} & x < 0 \\ x & \text{for} & x \ge 0 \end{cases}$	$f'(\alpha, x) = \begin{cases} f(\alpha, x) + \alpha & \text{for} & x < 0 \\ 1 & \text{for} & x \ge 0 \end{cases}$	$(-\alpha, \infty)$	C^1 when $\alpha = 1$, otherwise C^0

Activation functions approximate the sharp decision boundary.

Source: Wikipedia

How many functions can be modeled using a Perceptron?



Source: R. Rojas, Intro to Neural Networks

Vapnik-Chervonenkis Dimension

Definition 3.1 (VC Dimension [47]). The VC dimension D_{VC} of a hypothesis space f is the maximum integer $D = D_{VC}$ such that *some dataset* of cardinality D can be shattered by f. Shattered by f means that any arbitrary labeling can be represented by a hypothesis in f. If there is no maximum, it holds $D_{VC} = \infty$.

How many points can we label in general?

Formula by Schlaefli (1852):

$$T(n,k) = T(n-1,k) + T(n-1,k-1), \qquad (3)$$

where $T(n,1) = T(1,k) = 2$ or iteratively:

$$T(n,k) = 2\sum_{l=0}^{k-1} \binom{n-1}{l} \qquad (4)$$

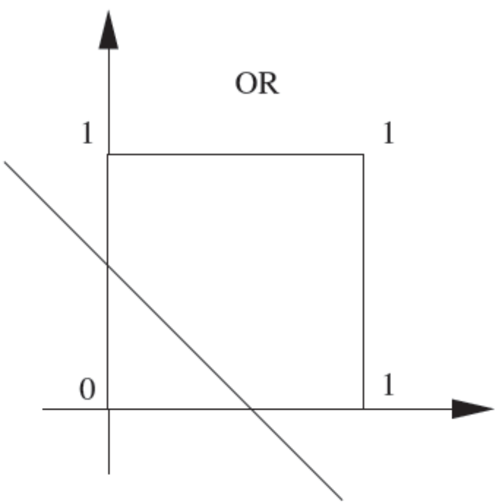
$n \backslash k$	1	2	3	4	5	6	7	8
1	**2**	2	2	2	2	2	2	2
2	2	**4**	4	4	4	4	4	4
3	2	6	**8**	8	8	8	8	8
4	2	8	14	**16**	16	16	16	16
5	2	10	22	30	**32**	32	32	32
6	2	12	32	52	62	**64**	64	64
7	2	14	44	84	114	126	**128**	128
8	2	16	58	128	198	240	254	**256**

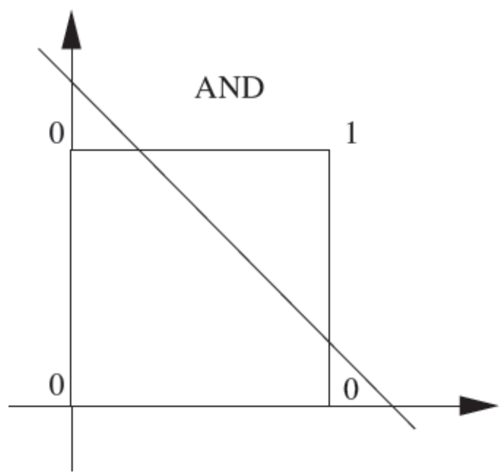
Table 1: Some values of the $T(n,k)$ function indicating the number of distinct threshold functions on n points in general position in k dimensions as defined by [22].

$$T(n,k) = 2^n \text{ for } k >= n.$$

Example: Boolean Functions

x_1	x_2	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

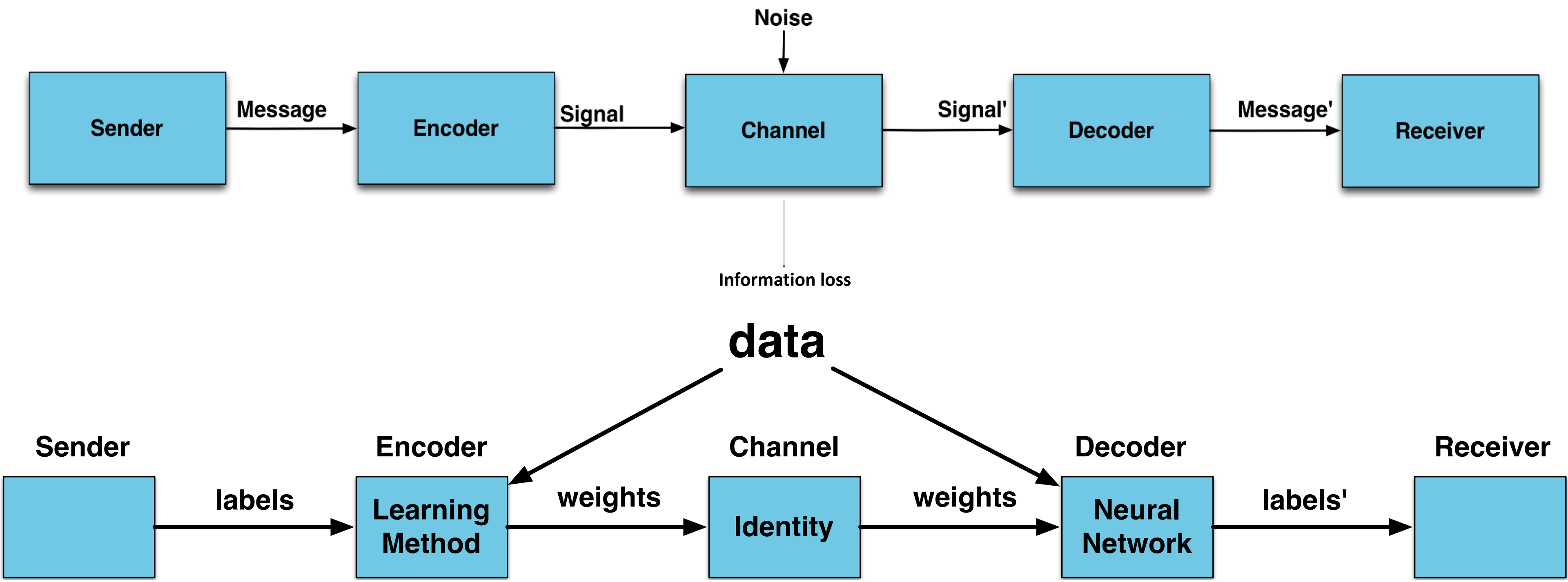




- 2^{2^v} functions of v boolean variables
- 2^v labelings of 2^v points.
- For $v=2$, all but 2 functions work: XOR, NXOR

Source: R. Rojas, Intro to Neural Networks

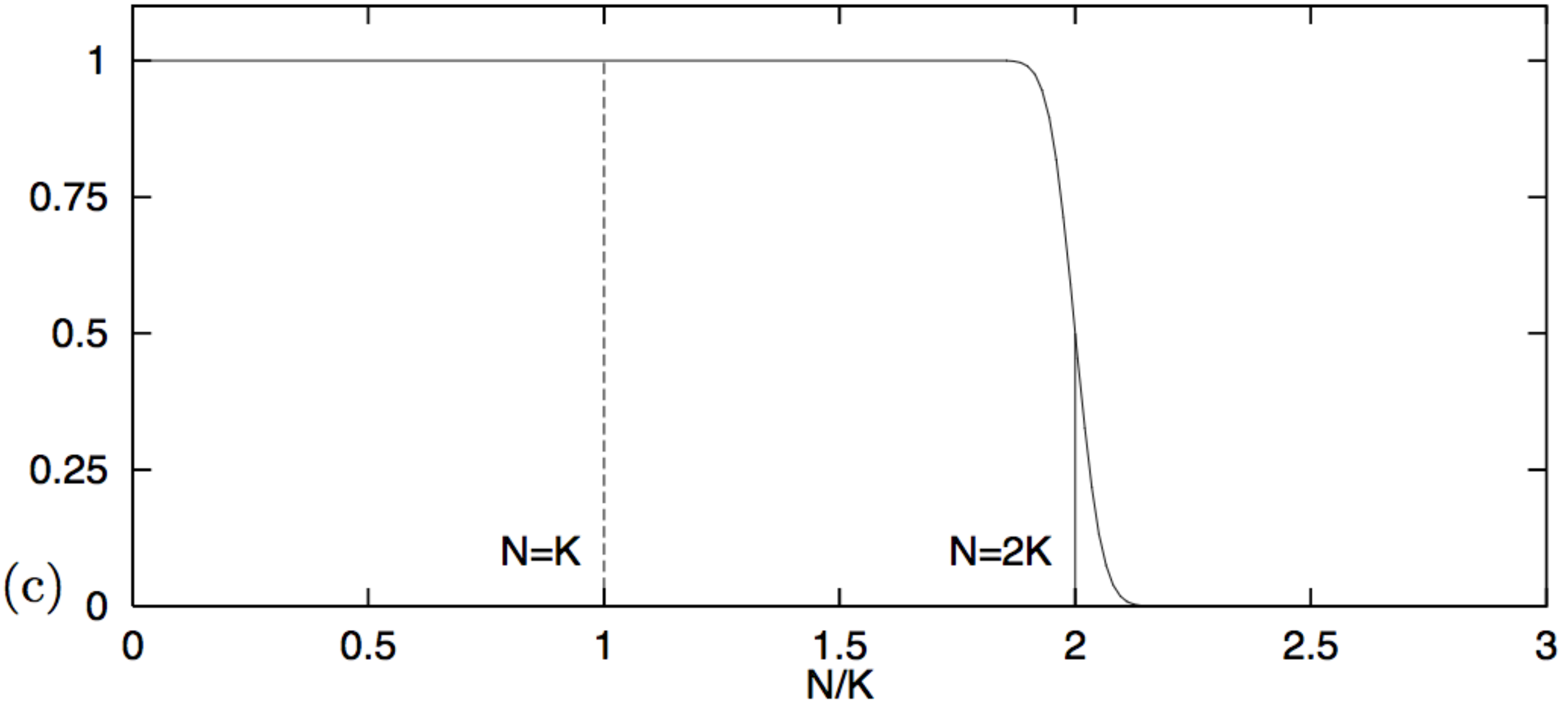
Machine Learning as an Encoder/Decoder



Main trick: Let the Machine Learner learn uniform random points!

Source: D. MacKay: Information Theory, Inference and Learning

Critical Points: Perceptron (Cover, MacKay)



N=K: VC Dimension (for points in random position)

N=2K: Cover/MacKay Capacity

Source: D. MacKay: Information Theory, Inference and Learning

Generalizing from Perceptron to Perceptron Networks

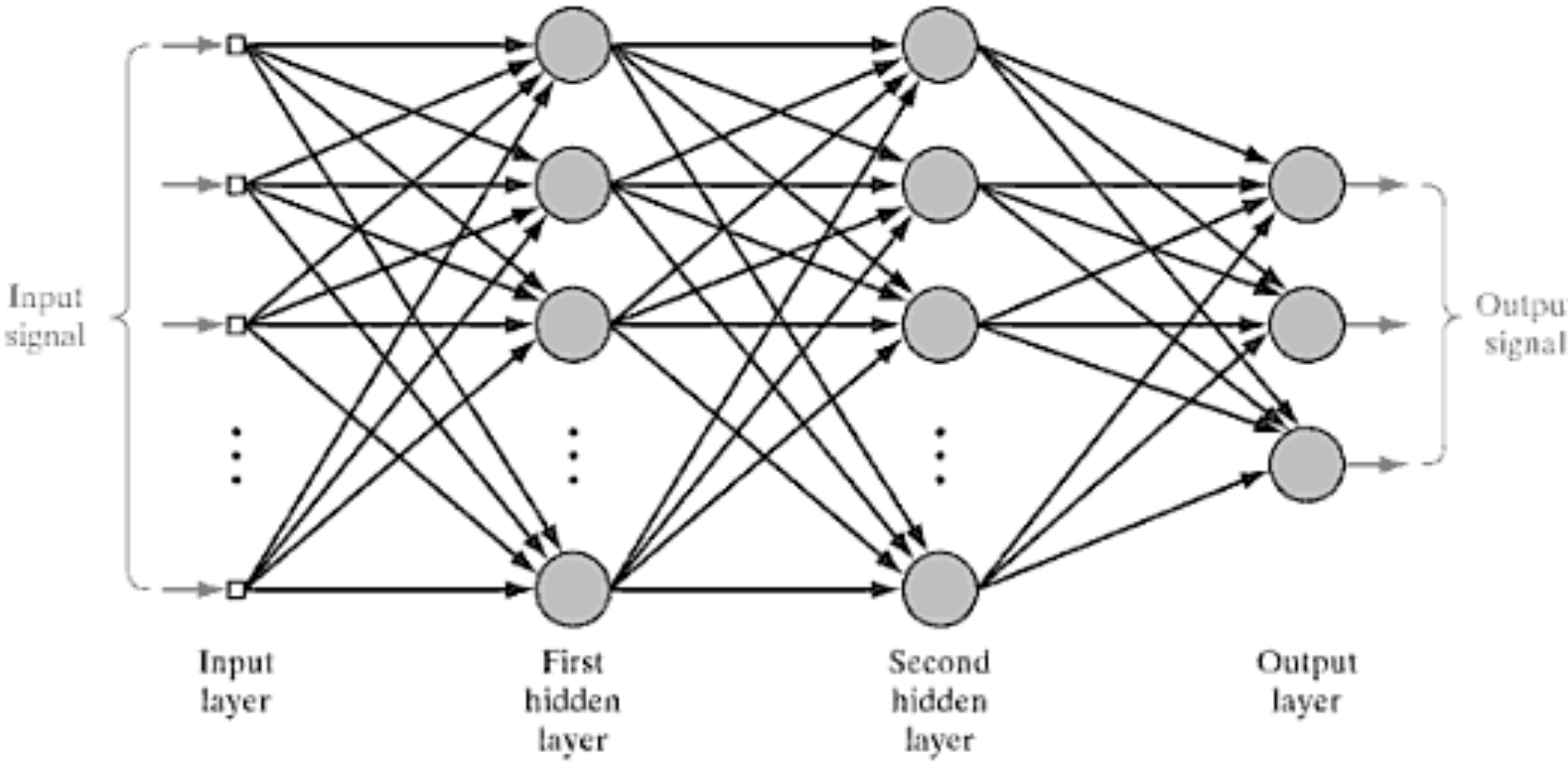
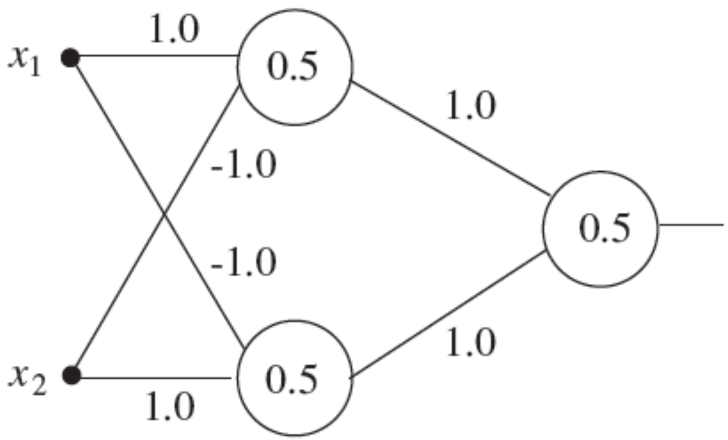


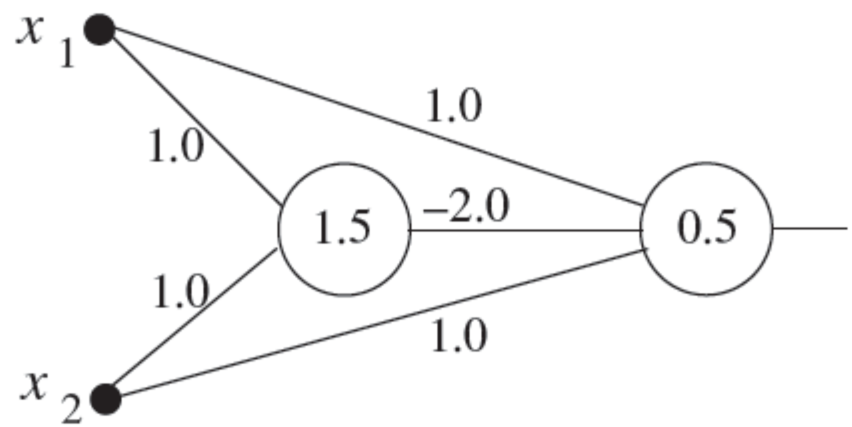
FIGURE 4.1 Architectural graph of a multilayer perceptron with two hidden layers.

Source: Wikipedia

Careful: Other Architectures



Typical MLP



Shortcut Network

Example Solutions to XOR

Source: R. Rojas, Intro to Neural Networks

Best Case Scenario?

Just measure in bits!

The *memory* capacity of **any binary classifier** cannot be better than the number of relevant bits in the model (pigeon hole principle, no universal lossless compression).

This is: *n* bits in the model can *maximally* model *n* bits of data.

Next Lecture

- Capacity for Neural Networks explained: See also cheat sheet.
- Practical applications
- Demo