Reproducibility and Experimental Design for Machine Learning

Gerald Friedland (UC Berkeley)

Paper, Demo, etc: https://tfmeter.icsi.berkeley.edu

About me….

- Adjunct Faculty, UC Berkeley
- Data Scientist at National Lab
- Started work in Machine Learning in 2001
Some Previous Work


http://www.siox.org

In GIMP, Krita, Inkscape, and other open source tools since 2005
The Multimedia Commons (YFCC100M)

100M videos and images, and a growing pool of tools for research with easy access through Cloud Computing

Collaboration Between Academia and Industry:

100.2M Photos
800K Videos

Features for Machine Learning
(Visual, Audio, Motion, etc.)

User-Supplied Metadata
and New Annotations

Tools for Searching,
Processing, and Visualizing

Supported in part by NSF Grant 1251276
“BIGDATA: Small: DCM: DA: Collaborative Research:
SMASH: Scalable Multimedia content Analysis in a High-level language”
Jupyter Integration

YFCC100M+MMCommons+Amazon’s MXNet

```python
statinfo = os.stat(filepath)
print('Successfully downloaded', imname, statinfo.st_size, 'bytes.')
return filepath
```

Now we can predict an image's location. We use images from Placing Task 2016 dataset to evaluate the result.

```python
imname = 'd661b83d659af4d818dd6edf54096.jpg'
result = predict_and_evaluate(imname)
```

('Ground truth: ', (36.016233, -114.737316))
rank=1, prob=0.462128, lat=36.0164119656, lng=-114.737289028, dist from groundtruth=0.020047 km
rank=2, prob=0.260691, lat=36.0141364684, lng=-114.733238705, dist from groundtruth=0.434543 km
rank=3, prob=0.236675, lat=36.0164436503, lng=-114.740746605, dist from groundtruth=0.309436 km
rank=4, prob=0.006943, lat=35.9723883996, lng=-113.79149647, dist from groundtruth=85.229922 km
rank=5, prob=0.006833, lat=36.8630814967, lng=-111.561140839, dist from groundtruth=299.302093 km
Start of this work: Simple Question

▪ **How much** money (cpu time, memory, IO) do I need to budget for my deep learning experiment?

▪ State of the Art: No answer. For example, ImageNet models vary significantly:
  ▪ AlexNet: 238MB model, 2.27Bn Ops
  ▪ DarkNet: 28MB model, 0.96Bn Ops
  ▪ VGG-16: 528 MB, 30.94Bn Ops

A game...

- Continue the sequence:
  - 2, 4, 6, 8, ....
  - 6, 5, 1, 4, ..... 

- What is the next number?
  - 100000 (sequence 1)
  - 100000 (sequence 2)

- Why?
The Scientific Method

Data Science: The Science of Automating the Scientific Method

The Scientific Method: Practical (traditional)

\[ E = mc^2 \]
The Scientific Method: Practical (new)

\[ E = mc^2 \]
Thought Framework: Reproducibility

An experimental result is not fully established unless it can be independently reproduced. [https://project.inria.fr/acmrmrreproducibility/](https://project.inria.fr/acmrmrreproducibility/)

- Repetition of experiment: Predict outcome X, observe outcome X.

- Reproduction of an experiment: Independent party recreates the experimental setup of an experiment and repeats it.

- Reproduction requires a complete description of all factors that make the experiment repeatable.

- Understanding: What are the minimum amount of factors that make the experiment repeatable and how do they influence each other?
Repetition, Reproduction, Understanding?

- Downloading the docker instance and re-running on your campus.
- Downloading the trained model and re-running it on the same data.
- Using the exact software versions and exact configuration of hyperparameters and retraining the same model to obtain similar results.
- Taking the description from a paper, rebuilding the setup as described and obtaining the same accuracy and adversarial examples.
- Taking the description from a paper, rebuilding the setup as described, obtaining the same accuracy and adversarial examples and explaining why a different setup yields other limits.
What factors make Machine Learning Repeatable, Reproducible, Understandable?

• Repeatable:  
  Set of all hyper parameters, seed, architecture, exact software versions of all libraries, exact order of training sample presentation, etc (?)

• Reproducible:  
  Capacity, perfect training (i.e. training that guarantees to reach a global minimum error).

• Understandable:  
  Minimum capacity, perfect training.
Thought Framework: Machine Learning

- Intelligence: *The ability to adapt* (Binet and Simon, 1904)

- Machine learning *adapts a finite state machine* $M$ *to an unknown function based on observations.*

- Input: $n$ rows of observations (instances) in a table with header:
  
  $$(x_1, x_2, \ldots, x_m, f(\vec{x}))$$

  where $f(\vec{x})$ is a column with labels we call target function.

- Output: State machine $M$ that maps a point
  
  $$(x_1, x_2, \ldots, x_m) \implies f(\vec{x})$$
Thought Framework: Machine Learning

Assume

\[ x_i \in \mathbb{R}, f(\vec{x}) \in \{0,1\} \] (binary classifier)

Question:

How many state transitions does \( M \) need to model the training data?

Refresh: Memory Arithmetic

• *Information is reduction of uncertainty:*
  \[
  H = -\log_2 P = -\log_2 \frac{1}{\text{#states}} = \log_2 \text{#states}
  \]
  measured in bits.

• Information: \(\log_2 \text{#states}\) (positive bits)
  Uncertainty: \(\log_2 P = \log_2 \frac{1}{\text{#states}}\) (negative bits)

• If states are not equiprobable, *Shannon Entropy* provides tighter bound.
  Math: Assumptions needed! (infinity, distribution)
  Engineering: Estimate using binning
Thought Framework: Machine Learning

Assume

\[ x_i \in \mathbb{R}, f(\vec{x}) \in \{0,1\} \]

(binary classifier)

Question:

How many state transitions does \( M \) need to model the training data?

Maximally: \#rows (lookup table)
Minimally: ? (Kolmogorov Complexity)
Thought Framework: Machine Learning

- **Intellectual Capacity:** The number of unique target functions a machine learner is able to represent (as a function of the number of model parameters).

- **Memory Equivalent Capacity (MEC):** A machine learner’s intellectual capacity is memory-equivalent to $N$ bits when the machine learner is able to represent all $2^N$ binary labeling functions of $N$ uniformly random inputs.

- At MEC or higher, $M$ is able to **memorize** all possible state transitions from the input to the output.
This Talk: Main trick

Memorization is worst-case generalization

• If we deduce nothing from data, the only thing we can do is memorize the observations verbatim.
• Using as many parameters as needed for memorization is therefore an indicator that the machine learner did not deduce anything (overfitting).
• Reducing parameters below memorization capacity will, in the best case, make the machine learner forget what’s not relevant with regards to the target function: generalization.
Memorization is worst-case generalization.

For binary classifiers:

\[
G = \frac{\text{correctly classified instances}}{\text{Memory Equivalent Capacity}} \quad \text{[bits / bit]}
\]

- \(G < 1\) \(\Rightarrow\) \(M\) needs more training/data (not even memorizing)
- \(G = 1\) \(\Rightarrow\) \(M\) is memorizing = overfitting
- \(1 < G < G_{MEM}\) \(\Rightarrow\) \(M\) could be implementing a lossless compression (and still overfit)
- \(G > G_{MEM}\) \(\Rightarrow\) \(M\) is generalizing (no chance for overfitting)
Maximum Lossless Compression

\[ G_{MEM} = \frac{\sum_{i=1}^{|\Sigma|} p_i \log_2 p_i}{\log_2 |\Sigma|} \]

\[ \Sigma = \text{Set of observations (rows in table)} \]

\[ p_i = \text{Probability to observe an element of } \Sigma \]
Generalization in Machine Learning

\[ G = \frac{\text{#correctly classified instances}}{\text{Memory Equivalent Capacity}} \times \left[ \frac{\text{bits}}{\text{bit}} \right] \]

Advantages of this definition:

- Keep current approach with training/validation/benchmark sets.
- No i.i.d. requirement for train/test set: Only requirement is input points are distinct!
- No distributional assumptions.
How do we calculate the Memory Equivalent Capacity?

- Binary Decision Tree: Depth of tree (if perfect).
- Neural Network (reminder of talk)
- Random Forrest: TBD
- SVN: TBD
- k-NN: TBD
- GMMs: TBD
Machine Learning as Engineering Discipline

• Supervised **Machine Learners have a Memory Equivalent Capacity in bits** that is **computable** and **measurable**.
  
  • Artificial Neural Networks with gating functions (Sigmoid, ReLU, etc.) have
    
    • a capacity upper limit that can be determined analytically using 4 principles
    
    • an effective capacity that can be measured on actual implementations.
  
• Predicting and measuring capacity allows for task-independent optimization of a concrete network architecture, learning algorithm, convergence tricks, etc...

• Capacity requirement can be approximately predicted given the input data and ground truth.
Repeat: The Perceptron

Physical interpretation: Energy threshold

\[ f(x) = \begin{cases} 
1 & \text{if } w \cdot x + b > 0 \\
0 & \text{otherwise} 
\end{cases} \]
**Repeat: Activation Functions (too many)**

<table>
<thead>
<tr>
<th>Function</th>
<th>Formula</th>
<th>Derivative</th>
<th>Domain</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>$f(x) = x$</td>
<td>$f'(x) = 1$</td>
<td>$(-\infty, \infty)$</td>
<td>$C^\infty$</td>
</tr>
<tr>
<td>Binary step</td>
<td>$f(x) = \begin{cases} 0 &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(x) = \begin{cases} 0 &amp; \text{for } x \neq 0 \ ? &amp; \text{for } x = 0 \end{cases}$</td>
<td>${0,1}$</td>
<td>$C^{-1}$</td>
</tr>
<tr>
<td>Logistic (a.k.a. Soft step)</td>
<td>$f(x) = \frac{1}{1 + e^{-x}}$</td>
<td>$f'(x) = f(x)(1 - f(x))$</td>
<td>$(0,1)$</td>
<td>$C^\infty$</td>
</tr>
<tr>
<td>TanH</td>
<td>$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$</td>
<td>$f'(x) = 1 - f(x)^2$</td>
<td>$(-1,1)$</td>
<td>$C^\infty$</td>
</tr>
<tr>
<td>ArcTan</td>
<td>$f(x) = \tan^{-1}(x)$</td>
<td>$f'(x) = \frac{1}{x^2 + 1}$</td>
<td>$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$</td>
<td>$C^\infty$</td>
</tr>
<tr>
<td>Softsign [7][8]</td>
<td>$f(x) = \frac{x}{1 +</td>
<td>x</td>
<td>}$</td>
<td>$f'(x) = \frac{1}{(1 +</td>
</tr>
<tr>
<td>Rectified linear unit (ReLU)[9]</td>
<td>$f(x) = \begin{cases} 0 &amp; \text{for } x &lt; 0 \ x &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(x) = \begin{cases} 0 &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$[0,\infty)$</td>
<td>$C^6$</td>
</tr>
<tr>
<td>Leaky rectified linear unit (Leaky ReLU)[10]</td>
<td>$f(x) = \begin{cases} 0.01x &amp; \text{for } x &lt; 0 \ x &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(x) = \begin{cases} 0.01 &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$(-\infty,\infty)$</td>
<td>$C^6$</td>
</tr>
<tr>
<td>Parameteric rectified linear unit (PReLU)[11]</td>
<td>$f(\alpha, x) = \begin{cases} \alpha x &amp; \text{for } x &lt; 0 \ x &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(\alpha, x) = \begin{cases} \alpha &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$(-\infty,\infty)$</td>
<td>$C^6$</td>
</tr>
<tr>
<td>Randomized leaky rectified linear unit (RReLU)[12]</td>
<td>$f(\alpha, x) = \begin{cases} \alpha x &amp; \text{for } x &lt; 0 \ x &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(\alpha, x) = \begin{cases} \alpha &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$(-\infty,\infty)$</td>
<td>$C^6$</td>
</tr>
<tr>
<td>Exponential linear unit (ELU)[13]</td>
<td>$f(\alpha, x) = \begin{cases} \alpha(e^x - 1) &amp; \text{for } x &lt; 0 \ x &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$f'(\alpha, x) = \begin{cases} f(\alpha, x) + \alpha &amp; \text{for } x &lt; 0 \ 1 &amp; \text{for } x \geq 0 \end{cases}$</td>
<td>$(-\alpha, \infty)$</td>
<td>$C^1$ when $\alpha = 1$, otherwise $C^0$</td>
</tr>
</tbody>
</table>

**Activation functions approximate the sharp decision boundary.**


Gerald Friedland, [http://www.gerald-friedland.org](http://www.gerald-friedland.org)
How many binary functions can one model using a single Perceptron?

Source: R. Rojas, Intro to Neural Networks
Example: Boolean Functions

\[ x_1 \quad x_2 \quad f_0 \quad f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \quad f_7 \quad f_8 \quad f_9 \quad f_{10} \quad f_{11} \quad f_{12} \quad f_{13} \quad f_{14} \quad f_{15} \]

| 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 |
| 0 1 | 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 |
| 1 0 | 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 |
| 1 1 | 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 |

- \( 2^v \) possible labelings of \( v \) boolean variables

- \( 2^v \) labelings of \( 2^v \) points.

- For \( v=2 \), all but 2 functions work: XOR, NXOR

Source: R. Rojas, Intro to Neural Networks
Machine Learning as an Encoder/Decoder

Sender → Encoder → Channel → Decoder → Receiver

Data

Sender → Encoder → Channel → Decoder → Receiver

Main trick: Let the Machine Learner label random points!

Source: D. MacKay: Information Theory, Inference and Learning
Critical Points: Perceptron (Cover, MacKay)

N=K: VC Dimension (for points in random position)
N=2K: Cover/MacKay Information Capacity

Source: D. MacKay: Information Theory, Inference and Learning

From a Perceptron to Perceptron Networks

FIGURE 4.1 Architectural graph of a multilayer perceptron with two hidden layers.

Careful: Other Architectures

Typical MLP

Shortcut Network

Example Solutions to XOR

Source: R. Rojas, Intro to Neural Networks
Solution: Calculate in bits!

Assume: $y_i, x_i \in \{0,1\}$, $x_i$ uniformly distributed
$n$ bits of memory: $f(x_1,\ldots,x_n)=x_1,\ldots,x_n$. (identity function).

Machine Learner:

binary classifier: $f(x_1,\ldots,x_n)=y_1$
multi-class/regression: $f(x_1,\ldots,x_n)=y_1,\ldots,y_m$

Memory Equivalent Capacity: The number of configurations of uniformly distributed $x_1,\ldots,x_n$ that a machine learner can guarantee to label correctly.
Memory Equivalent Capacity for Neural Networks

1) The output of a perceptron is maximally 1 bit.
2) The maximum memory capacity of a perceptron is the number of parameters (including bias) in bits.  
   (MacKay 2003)
3) The maximum memory capacity of perceptrons in a neural network is additive, except 4.  
   (MacKay 2003 speculative, Friedland and Krell 2017)
4) The maximum memory capacity of a layer of perceptrons depending on a previous layer of perceptrons is limited by the maximum output (in bits) of the previous layer.  
   (Data Processing Inequality, Tishby 2012)
Examples: How many bits of maximal capacity?

3 bits

2*3 bits + \text{min}(2,3) \text{ bits} = 8 \text{ bits}
Examples: How many bits of maximal capacity?

2*3 bits + \(\min(2,2\times3)\) bits + \(\min(2,3)\) bits = 10 bits
Examples: How many bits of maximal capacity?

3 bits + 4 bits = 7 bits

Shortcut or ResNet
Characteristic Curve of a Theoretical 3-Layer MLP

![Graph showing the characteristic curve of a theoretical 3-layer MLP.](Graph)

- **Perfect classification accuracy**: $T(n,k)/2^n$
- **Input dimension k**: 2, 4, 8, 16, 32, 64, 128
- **Number of samples n divided by k**

Gerald Friedland, [http://www.gerald-friedland.org](http://www.gerald-friedland.org)
Characteristic Curve of an Actual 3-Layer MLP

Python scikit-learn, 3-Layer MLP
Predicting Capacity Requirements

Given data and labels: How much actual capacity do I need to memorize the function?

Idea:
1) Worst case: Let’s build a memorization network where only the biases are trained
2) Expected case: How much parameter reduction can (exponential) training buy us?
data: array of length $i$ containing vectors $x$ with dimensionality $d$
lables: a column containing 0 or 1
$MaxCapReq(data, labels)$
thresholds ← 0
loop over $i$: $table[i] ← (\sum x[i][d], label[i])$
sortedtable ← sort(table, key = column 0)
class ← 0
loop over $i$: if not sortedtable[i][1] == class then
    class ← sortedtable[i][1]
    thresholds ← thresholds + 1
end
maxcapreq ← thresholds * $d + thresholds + 1$
expcapreq ← $\log_2(thresholds + 1) \times d$
print ”Max: ” + maxcapreq + ” bits”
print ”Exp: ” + expcapreq + ” bits”

“Dumb” Network

Runtime: $O(n \log n)$
Predicting Expected Minimum Memory Equivalent Capacity

Dumb Network:
- Highly inefficient.
- Potentially not 100% accurate (hash collisions).
- We can assume training weights (and biases) gets 100% accuracy while reducing parameters.

Expected Reduction: Exponential!

$n$ thresholds should be able to be represented with $\log_2 n$ weights and biases (search tree!).
## Empirical Results

All results repeatable at: [https://github.com/fractor/nntailoring](https://github.com/fractor/nntailoring)

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Max Capacity Requirement</th>
<th>Expected Capacity Requirement</th>
<th>Validation (% accuracy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AND, 2 variables</td>
<td>4 bits</td>
<td>2 bits</td>
<td>2 bits (100%)</td>
</tr>
<tr>
<td>XOR, 2 variables</td>
<td>8 bits</td>
<td>4 bits</td>
<td>7 bits (100%)</td>
</tr>
<tr>
<td>Separated Gaussians (100 samples)</td>
<td>4 bits</td>
<td>2 bits</td>
<td>3 bits (100%)</td>
</tr>
<tr>
<td>2 Circles (100 samples)</td>
<td>224 bits</td>
<td>12 bits</td>
<td>12 bits (100%)</td>
</tr>
<tr>
<td>Checker pattern (100 samples)</td>
<td>144 bits</td>
<td>12 bits</td>
<td>12 bits (100%)</td>
</tr>
<tr>
<td>Spiral pattern (100 samples)</td>
<td>324 bits</td>
<td>14 bits</td>
<td>24 bits (98%)</td>
</tr>
<tr>
<td>ImageNet: 2000 images in 2 classes</td>
<td>906984 bits</td>
<td>10240 bits</td>
<td>10253 bits (98.2 %)</td>
</tr>
</tbody>
</table>
Training

- Everything we did so far assumes perfect training. This is, training that guarantees to reach the global minimum error.
- Perfect training requires exponential time.
- Imperfect training means Memory Equivalent Capacity is effectively reduced.
- How to measure that: ?
From Memorization to Generalization

Memorization is worst-case generalization.

Good news:
• Real-world data is not random.
• The information capacity of a perceptron is usually >1 bit per parameter (Cover, MacKay).

This means, we should be able to use less parameters than predicted by memory capacity calculations.
Suggested Engineering Process for Generalization

- Start at approximate expected capacity.
- Train to >98% accuracy. If impossible, increase parameters.
- Retrain iteratively with decreased capacity while testing against validation set.
  Should see: decrease in training accuracy with increase in validation set accuracy
- Stop at minimum capacity for best held-out set accuracy.

Best case scenario: As parameters are reduced, neural network fails to memorize only the insignificant (noise) bits.
Generalization Process: Expected Curve
Overcapacity Machine Learning: Issues

- Waste of money, energy, and time. Bad for environment.

- The less parameters => the better the generalization rule => the higher adaptation per parameter => the higher the chance an unseen instance can be predicted correctly.

Reminder: Occam’s Razor

Among competing hypotheses, the one with the fewest assumptions should be selected.

For each accepted explanation of a phenomenon, there may be an extremely large, perhaps even incomprehensible, number of possible and more complex alternatives, because one can always burden failing explanations with ad hoc hypotheses to prevent them from being falsified; therefore, simpler theories are preferable to more complex ones because they are more testable.

(Wikipedia, Sep. 2017)
General Generalization

- Binary classifier (repeat):

  \[ G = \frac{\text{#correctly classified instances}}{\text{Memory Equivalent Capacity}} \quad [\text{bits/bit}] \]

- Multi-class/regression:

  \[ G = \frac{\text{#correctly classified instances}}{\text{#instances that can be memorized}} \]
Non-Statistical Definition (Literature)

Informally: When do two different inputs lead to the same machine learner output. This is, which bits can be ignored in the comparison.

Statistical equivalent: How many bits per bit can be ignored on average (see $G$ measure).
Central Hypothesis

Given perfect training, capacity makes a machine learning experiment reproducible independent of other hyperparameters.

- This is, initialization, learning rate, batch size, and architecture do not matter!
- TODO: Improve training!
Demo: Experimental Design for TensorFlow

http://tfmeter.icsi.berkeley.edu

Exercises (joint)

Please go to

https://tinyurl.com/y3xn8dnf

and work on the homework 1, 2, 3, 4, 5, and 6 in groups.

We will discuss them in about 45 min.