How fast can we solve problems?

- **P**
Polynomial Time
How do we define running time?

\[ \text{TIME}(t(n)) \]

All languages decidable by some TM in \( O(t(n)) \) steps.

\[ P = \bigcup_{c \geq 0} \text{TIME}(n^c) \]

What about non-determinism?
**NTIME**(\(t(n)\))

All languages decidable by some non-deterministic TM in \(O(t(n))\) steps.

**NP**

Non-deterministic Polynomial Time

\[\text{NP} = \bigcup_{c \geq 0} \text{NTIME}(n^c)\]

NP languages are polynomially verifiable.
**Definition.** Let $V((x,y))$ be a TM. $V$ *polynomially verifies* a language $L$ if $V((x,y))$ runs in deterministic polynomial time and if, for all $x \in \Sigma^*$:

1. $x \in L \implies \exists y \in \Sigma^*$ such that $V((x,y))$ accepts.
2. $x \notin L \implies \forall y \in \Sigma^*$ it is the case that $V((x,y))$ rejects.

$L$ is polynomially verifiable if there exists a polynomial-time TM that verifies it.

**Proposition.** $L \in \text{NP} \iff L$ is polynomially verifiable.

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$3\text{-Sat} = \{ \langle \phi \rangle \mid \text{there exists a satisfying assignment of 3-CNF formula } \phi \}$

$\text{SAT} = \{ \langle \phi \rangle \mid \text{there exists a satisfying assignment of logical formula } \phi \}$

Decision vs. search
How do I find some satisfying assignment?

$L \in \text{NPH}$ if $\forall A \in \text{NP}, A \leq_p L$

$L \in \text{NPC} \iff L \in \text{NP}$ and $L \in \text{NPH}$
\( \text{Sat} \in \text{NPC} \)

\( 3\text{-Sat} \in \text{NPC} \)

\[ \text{CLIQUE} = \{ \langle G, k \rangle \mid G \text{ is a graph that contains a clique } k \text{ vertices} \} \]

\[ \text{VERTEX-COVER} = \{ \langle G, k \rangle \mid G \text{ is a graph that contains a subset of } k \text{ vertices that cover } G\text{'s edges} \} \]

Reading: Sipser 7.2-7.4