Viruses

A Turing machine that replicates itself.
q(w):
1. Build a TM $P_w$:
   1. Ignore the input.
   2. Erase the tape.
   3. Write $w$ and halt.

2. Return $<P_w>$. 

TM B(w):
1. Compute $<P_w>$. 
2. Print $<P_w>, <w>$. 

What if we call B($<B>$)? 

TM SELF
\langle SELF \rangle = \langle AB \rangle

Let \( T(x, w) \) be any TM. We can build a TM \( R(w) = T(\langle R \rangle, w) \).

\begin{align*}
\text{The Recursion Theorem} \\
\text{We want:} \\
\text{SELF} = P_{\langle B \rangle}<B>.
\end{align*}

\begin{align*}
\text{We want:} \\
R &= P_{\langle BT \rangle}<B><T>.
\end{align*}
\[ R = \langle ABT \rangle \]

TM B(x,w):
1. Compute A = \langle P_x \rangle.
2. Print \langle P_x \rangle, \langle x \rangle, \langle w \rangle.

What if we call B(<BT>)?

Example 1:
T(x,w):
1. Output x,w.
2. Halt.

Example 2:
T(x,w):
1. Output x.
2. Halt.
Example 3:
\[ T(x,w): \]
1. Run \( x(w) \) if \( x \) is a TM.

Example 4:
\[ T(x,<M>): \]
1. Run \( M(x) \) if \( M \) is a TM.

An Undetectable Virus

Suppose \( \text{Detect(<V>)} \) accepts if \( V \) is a virus.
Unhappy Meals by Michael Pollan
The New York Times Magazine
January 28, 2007

Eat food. Not too much. Mostly plants.

That, more or less, is the short answer to the supposedly incredibly complicated and confusing question of what we humans should eat in order to be maximally healthy. I hate to give away the game right here at the beginning of a long essay, and I confess that I'm tempted to complicate matters in the interest of keeping things going for a few thousand more words. I'll try to resist but will go ahead and add a couple more details to flesh out the advice. Like: A little meat won't kill you, though it's better approached as a side dish than as a main. And you're much better off eating whole fresh foods than processed food products. That's what I mean by the recommendation to eat "food." Once, food was all you could eat, but today there are lots of other edible foodlike substances in the supermarket. These novel products of food science often come in packages festooned with health claims, which brings me to a related rule of thumb: if you're concerned about your health, you should probably avoid food products that make health claims. Why? Because a health claim on a food product is a good indication that it's not really food, and food is what you want to eat.

Uh-oh. Things are suddenly sounding a little more complicated, aren't they? Sorry. But that's how it goes as soon as you try to get to the bottom of the whole vexing question of food and health. Before long, a dense cloud bank of confusion moves in. Sooner or later, everything solid you thought you knew about the links between diet and health gets blown away in the gust of the latest study.
Eat food. Not too much. Mostly plants.


Cognitive load

Description of a string $x$: $<M,w>$ where $M$ is a TM and $M(w)$ outputs $x$. 
Complexity of a string $x$:

The length of the shortest $<M,w>$, $d(x)$.

\[ K(x) = |d(x)| \]

There exists a $c$ such that for all $x$, $K(x) \leq |x| + c$

There exists a $c$ such that for all $x$, $K(xx) \leq K(x) + c$
Compressibility

How much shorter than a string is its minimal description?

Not every string can be compressed.

Distinguishability of $x$.

The smallest TM $M$ such that:
1. $M(x)$ accepts.
2. $M(z)$ rejects for $z \neq x$
3. $M(f)$ runs in time bounded in $|f|$

Cryptographic Indistinguishability:

An adversary asks to encrypt two messages. One is encrypted and returned. The scheme is secure under chosen-plaintext attack if it is hard for the adversary to tell which was encrypted.
Reading: Sipser 6.1, 6.4