Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

\[ \Sigma = \{0, 1\} \]

What makes a language

Non

Regular
DFAs/NFAs cannot store information.

Suppose you have a stack on which to store information.

How might you recognize $L$?

$L = \{0^n1^n \mid n \geq 0\}$

$\Sigma = \{0, 1\}$
NFAs

\[ Q \] set of states

\[ \Sigma \] the alphabet

\[ \delta : Q \times \Sigma \rightarrow P(Q) \] transition function

\[ q_0 \in Q \] start state

\[ F \subseteq Q \] set of final states
Pushdown automata: push and pop a stack.

### NPDAs

- **$Q$** set of states
- **$\Sigma, \Gamma$** input, stack alphabet
- **$Q \times \Sigma^\epsilon \times \Gamma^\epsilon \rightarrow P(Q \times \Gamma^\epsilon)$** transition function
- **$q_0 \in Q$** start state
- **$F \subseteq Q$** set of final states

### Example

**Input and Stack Alphabet**

- **$\Sigma = \{0, 1\}$**
- **$\Gamma = \{0, \$\}**

**Language**

- **$L = \{0^n1^n \mid n \geq 0\}$**

**Transition Rules**

- **$0, \epsilon \rightarrow 0$**
- **$\epsilon, \epsilon \rightarrow \$**
- **$1, 0 \rightarrow \epsilon$**
- **$\epsilon, \epsilon \rightarrow \$**
- **$1, 0 \rightarrow \epsilon$**
- **$\epsilon, \$ \rightarrow \epsilon$**
What NPDA recognizes:

$L = \{0^i1^j2^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$

$\Sigma = \{0, 1, 2\}$

Build an NPDA that recognizes:

$L = \{ww^R\}$

$\Sigma = \{0, 1\}$
Build an NPDA that recognizes:
$L = \{ w \mid w \text{ has the same number of 0s and 1s} \}$

$\Sigma = \{ 0, 1 \}$
How can we define an NPDA’s language?

What CFG generates this language?

$L = \{0^n1^n \mid n \geq 0\}$

$\Sigma = \{0, 1\}$

Rules:

$A \rightarrow 0A1$

$A \rightarrow B$

$B \rightarrow \varepsilon$
Grammar $G$:

$A \to 0A1$

$A \to B$

$B \to \varepsilon$

$L(G) = \{0^n1^n \mid n \geq 0\}$

Derivation of a string:

$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000B111 \Rightarrow 000111$

What CFG generates this language?

$L = \{ww^R\}$

$\Sigma = \{0, 1\}$

Grammar $G$:

$A \to 0A0 \mid 1A1$

$A \to \varepsilon$
What CFG generates this language?

$L = \{w \mid w \text{ has the same number of 0s and 1s}\}$

$\Sigma = \{0, 1\}$

$L = \{w \mid w \text{ has the same number of 0s and 1s}\}$

**Grammar $G$:**

- $A \rightarrow 0A1A \mid 1A0A$
- $A \rightarrow \varepsilon$

Reading: Sipser 2.1, 2.2