MISC

Homework 1 due
Quiz 1 out Monday
(covers Sipser 1.1-1.3
self-timed, 2 hours)
What does it mean for a language to be REGULAR?
Is the following language regular?

\[ L = \{ w \mid w \text{ has the same number of } 01\text{s as } 10\text{s} \} \]

\[ \Sigma = \{ 0, 1 \} \]
\( L = \{ w \mid w \text{ has the same number of 01s as 10s} \} \)
Is the following language regular?

\[ L = \{ w \mid w \text{ has the same number of 0s and 1s} \} \]

\[ \Sigma = \{ 0, 1 \} \]
NON

Regular Languages
How do we prove that a language is not regular?
How to show a language $L$ is not regular:

1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.
Is the following language regular?

\[ L = \{a^n b^n \} \]
How to show a language $L$ is not regular:

1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.
Pumping Lemma
((0 \cup 1)*0)^*
\(((0 \cup 1)^* 0)^*\)
Let’s look at strings $s$, $|s| = 4$

What is true about the automaton for all strings of length 4 it reads?
What is the shortest string that will cause a repeated state?
Suppose DFA $M$ has $|Q|$ states. Any string $s$, $|s| \geq |Q|$ will cause $M$ to repeat a state.
What does it mean to repeat a state?
Input: 10
State Sequence: ABA

First appearance

Last appearance
Input: 10
State Sequence: ABA
Input: 10
State Sequence: ABA
Also accepted:
1010
101010
101010
Input: 010
State Sequence: ABBC
Also accepted: 01*0
Generalized process:
1. Pick a string $s$ of length $|Q|$.
2. Find where it repeats a state.
3. Repeat that part of the string.

$s = xyz$, where $y$ is the repeating part.
How to show a language $L$ is not regular:

1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.

(Draft 1)

Property: Every regular language with a DFA of $|Q|$ states has a string $s$ of length $|Q|$ where $s = xyz$ and $y$ can be repeated.
How to show a language L is not regular:
1. Identify some property P that is true for all regular languages.
2. Assume P holds true for language L.
3. Obtain a contradiction, thereby showing L is not regular.

(Draft 2)
Property: Every regular language with a DFA of |Q| states has a string s of length |Q| where s = xyz and y can be repeated and |y| > 0.
How to show a language $L$ is not regular:

1. Identify some property $P$ that is true for all regular languages.
2. Assume $P$ holds true for language $L$.
3. Obtain a contradiction, thereby showing $L$ is not regular.

(Draft 3)

Property: Every regular language with a DFA of $|Q|$ states has a string $s$ of length $|Q|$ where $s = xyz$ and $y$ can be repeated, $|y| > 0$, and $|xy| \leq |Q|$.
Pumping Lemma

For every regular language $L$ there exists some integer $p$ where for every string $s$ in $L$ of length at least $p$, $s = xyz$ and $y$ can be repeated, $|y| > 0$, and $|xy| \leq p$. 
How to show a language \( L \) is not regular:

1. Identify some property \( P \) that is true for all regular languages.
2. Assume \( P \) holds true for language \( L \).
3. Obtain a contradiction, thereby showing \( L \) is not regular.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

\[ \Sigma = \{0, 1\} \]
Is the following language regular?

\[
L = \{0^n1^n \mid n \geq 0\}
\]

1. Identify some property \( P \) that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string \( s \) of length \( p \) for which we can apply the pumping lemma.

2. Assume \( P \) holds true for language \( L \).
Is the following language regular?

\[ L = \{ 0^n1^n \mid n \geq 0 \} \]

1. Identify some property P that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string \( s \) of length \( p \) for which we can apply the pumping lemma.

2. Assume P holds true for language \( L \).

Assume \( L \) is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing \( L \) is not regular.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

1. Identify some property P that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string s of length p for which we can apply the pumping lemma.

2. Assume P holds true for language L.

Assume L is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing L is not regular.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]
Is the following language regular?

\[ L = \{ 0^n1^n \mid n \geq 0 \} \]

Let \( s = 0^p1^p \), \( s \in L \).
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \( s = 0^p1^p \ s \in L \)

The pumping lemma guarantees:
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \( s = 0^p1^p \) \( s \in L \)

The pumping lemma guarantees:

\[ s = xyz \quad xy^*z \in L \]
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \( s = 0^p1^p \) \( s \in L \)

The pumping lemma guarantees:

\[ s = xyz \quad xy^*z \in L \]

3 CASES
Is the following language regular?

\[ L = \{ 0^n 1^n \mid n \geq 0 \} \]

Let

\[ s = 0^p 1^p \quad s \in L \]

The pumping lemma guarantees:

\[ s = xyz \quad xy^*z \in L \]

3 CASES

1. \( y \in \{0^*\} \)
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \( s = 0^p1^p \) \( s \in L \)

The pumping lemma guarantees:

1. \( y \in \{0^*\} \)
2. \( y \in \{1^*\} \)
3 CASES

1. \( y \in \{0^*\} \)
2. \( y \in \{1^*\} \)
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

Let \[ s = 0^p1^p \quad s \in L \]

The pumping lemma guarantees:

1. \( y \in \{0^*\} \)
2. \( y \in \{1^*\} \)
3. \( y \in \{0^i1^j\} \)
Is the following language regular?

\[ L = \{ 0^n 1^n \mid n \geq 0 \} \]

\[ s = xyz \quad xy^*z \in L \]

3 CASES
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

3 CASES

1. \( y \in \{0^*\} \)
   Impossible because repeating \( y \) would produce more 0s than 1s.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

\[ s = xyz \quad xy^*z \in L \]

3 CASES

1. \( y \in \{0^*\} \)
   Impossible because repeating \( y \) would produce more 0s than 1s.

2. \( y \in \{1^*\} \)
   Impossible because repeating \( y \) would produce more 1s than 0s.
Is the following language regular?

\[ L = \{0^n1^n \mid n \geq 0\} \]

\[ s = xyz \quad xy^*z \in L \]

3 CASES

1. \( y \in \{0^*\} \) Impossible because repeating \( y \) would produce more 0s than 1s.

2. \( y \in \{1^*\} \) Impossible because repeating \( y \) would produce more 1s than 0s.

3. \( y \in \{0^i1^j\} \) Impossible because repeating \( y \) would mis-order 1s and 0s.
Therefore

\[ L = \{0^n1^n \mid n \geq 0\} \]

is NOT regular.
Is the following language regular?

$L = \{ w \mid w \text{ has the same number of 0s and 1s}\}$

$\Sigma = \{0, 1\}$
Pumping Lemma

For every regular language $L$ there exists some integer $p$ where for every string $s$ in $L$ of length at least $p$, $s = xyz$ and $y$ can be repeated, $|y| > 0$, and $|xy| \leq p$.

$L = \{ w \mid w \text{ has the same number of 0s and 1s} \}$
Is the following language regular?

$L = \{ w \mid w \text{ has the same number of 0s and 1s} \}$

1. Identify some property $P$ that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string $s$ of length $p$ for which we can apply the pumping lemma.

2. Assume $P$ holds true for language $L$.

Assume $L$ is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing $L$ is not regular.

$s = 0^p 1^p$

$|xy| \leq p \implies y \in \{0^*\} \implies xy^2z \notin L$
Is the following language regular?

$L = \{ \text{ww}^R \}$

$\Sigma = \{0, 1\}$
Pumping Lemma

For every regular language $L$ there exists some integer $p$ where for every string $s$ in $L$ of length at least $p$, $s = xyz$ and $y$ can be repeated, $|y| > 0$, and $|xy| \leq p$.

$L = \{ww^R\}$
Is the following language regular?

\[ L = \{ww^R\} \]

1. Identify some property P that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string s of length p for which we can apply the pumping lemma.

2. Assume P holds true for language L.

Assume L is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing L is not regular.

\[ s = 0^p110^p \]

\[ |xy| \leq p, \ |y| > 0 \implies y \in \{0^+\} \implies xy^0z \notin L \]
Is the following language regular?

\[ L = \{0^i1^j \mid i > j\} \]
\[ \Sigma = \{0, 1\} \]
Pumping Lemma

For every regular language $L$ there exists some integer $p$ where for every string $s$ in $L$ of length at least $p$, $s = xyz$ and $y$ can be repeated, $|y| > 0$, and $|xy| \leq p$.

$L = \{0^i1^j \mid i > j\}$
Is the following language regular?

$$L = \{0^i1^j \mid i > j\}$$

1. Identify some property P that is true for all regular languages.

There exists some DFA that recognizes any regular language and accepts some string s of length p for which we can apply the pumping lemma.

2. Assume P holds true for language L.

Assume L is regular and thus has a DFA and is pumpable.

3. Obtain a contradiction, thereby showing L is not regular.

$$s = 0^{p+1}1^p$$

$$|xy| \leq p, |y| > 0 \implies y \in \{0^+\} \implies xy^0z \notin L$$
Is the following language regular?

\[ L = \{0^i1^j0^k \mid i > 10 > j > k > 0\} \]

\[ \Sigma = \{0, 1\} \]
Is the following language regular?

\[ L = \{0^i 1^j 0^k \mid i > 10 > j > k > 0\} \]

\[ R = 0^{+}0^{10}((1^9(0^8 \cup 0^7 \cup \ldots \cup 0^1))\cup (1^8(0^7 \cup 0^6 \cup \ldots \cup 0^1))\cup \ldots \cup (1^2(0^1))) \]
Reading: Sipser 1.4