Due: April 30, 2010, 1:00pm

1. **Ford-Fulkerson (20 points).** (Re)prove the correctness of the Ford-Fulkerson max-flow algorithm.

2. **Ford-Fulkerson (20 points).** Give an example flow graph for which the Ford-Fulkerson algorithm never terminates. Your example may use non-integer values.

3. **Min cut (20 points).** Consider the following algorithm to find the min cut of a flow graph. First pick an arbitrary vertex \( v \in V \) and initialize \( S = \{v\} \). While it is possible to move a vertex from \( S \) to \( V - S \), or a vertex from \( V - S \) to \( S \), so as to reduce the cost of the cut (and to keep \( S \) and \( V - S \) non-empty), move the vertex. When no such move is possible, return the current cut. Prove that this algorithm is correct or give a counterexample.

4. **Min cut (10 points).** Give an example flow graph that has a unique min cut.

5. **Min cut (10 points).** Give an example flow graph that has more than one min cut.

6. **Min cut (20 points).** Give a polynomial-time algorithm that determines whether the min cut in a given flow graph is unique, and if not, finds at least two distinct min cuts.