Robust direction of arrival estimation

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ICSI Speech Group Lunch Talk
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<td>• Motivation, background and applications</td>
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<td>• Robustness</td>
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Motivation

MY THEORY IS THAT CONSCIOUSNESS IS THE ABILITY TO PREDICT AND THEN OBSERVE THE RESULTS OF ACTIONS.

SO I THINK YOU COULD BUILD A COMPUTER THAT WOULD BE FULLY CONSCIOUS.

OBVIOUSLY YOU'D NEED AN ARRAY OF SENSORS TO COLLECT THE DATA.
Motivation

- Recent (last 20 years or so) developments have made required hardware available at reasonable cost

- Jim Gray (1998 ACM Turing Award Winner):
  3. Hear as well as a person
  8. Remember what is seen and heard and quickly return it on request.
  10. Build a system that, given sounds, can answer questions about the sounds and summarize them as quickly and precisely as a human expert.
  12. Simulate being some other place as an observer and a participant.
Applications

- Seismic measurements, localization of earthquake epicenters
- Medical devices, especially ultrasonic imaging
- Telecommunications (e.g. localization of a mobile phone)
- Sonars, ships, boats, submarines
- Radars, airplanes, flight control, ships
- Infrasonics, CTBT nuclear test ban surveillance
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**Acoustic applications**

- Multimedia applications, multichannel spatial recordings
- Pre-processors: video conferences, automated cameras
- Advanced acoustic (noise) measurements and research
- Automatic surveillance (harbors, large storage areas, peace keeping)
- Illegal arms detection
Acoustic localization: Application examples


Image from Blumrich, Altmann : Medium-range localisation of aircraft via triangulation, ©JASA 2000
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Speech applications

- Speech and sound enhancement
- Hands-free systems
- Speech recognizers
- Hearing aids
- Meeting rooms
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Spatial signals

- Signals:
  \[ s(x, t) \quad , \quad x = [x, y, z]^T \]

- Propagating waves:
  \[ \nabla^2 s(x, t) = \frac{1}{c^2} \frac{\partial^2 s(x, t)}{\partial t^2} \]

- Planar wave solution

- Spherical wave solution
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Spatial signals

- Observations at isolated points

- Resulting signals:
  \[ s_i(t) = s(x_i, t + \tau_i) \]

- Relation between sensors (slightly ideal...)
  \[ s_i(t) = s_j(t + \tau_{ij}) \]

- Sensor and source locations, and shape of wavefront determine \( \tau_{ij} \)
  
  → Time delays, array, and wavefront assumption give the source "location"
Localization prototype

Sensors (mics) → Pre-processing → Time delay estimation → Direction of arrival estimation → Localization

Sometimes the last two steps are difficult to separate
Time delay estimation

- Problem generally solved in the 70’s
- No "silver bullet", optimal $\leftrightarrow$ practical
- Usually utilize some form of "alignment", or correlation:

$$\hat{\tau}_{(i,j)} = \arg\max_{\tau} \sum_{t=0}^{N-1} s_i(t) s_j(t + \tau)$$

- With speech, reverberations are a problem
  $\leftarrow$ GCC-PHAT has been proposed to be robust
  - Major improvements not very likely
DOA estimation - spherical waves

- Hyperbolic equations
- Practical situations include noise and errors
  - No (accurate) closed form solution
- Iterative solutions ← time-consuming
- Distance of source affects the solution, initial guess needed
- If successful, solution gives DOA and location
- Requires propagation speed a priori
DOA estimation - planar waves

- Time delay

$$\tau_{(i,j)} = \frac{d_{(i,j)} \cos(\vartheta)}{c}$$

- Angle of arrival

$$\vartheta = \cos^{-1} \frac{\tau_{(i,j)} c}{d_{(i,j)}}$$

- Requires propagation speed
DOA estimation - planar waves

\[ \tau_{(i,j)} = \frac{1}{c} d_{(i,j)} \cos(\vartheta) \] is actually

\[ \tau_{(i,j)} = \| k \| \| x_{(1,2)} \| \cos(\vartheta) \]

\[ = k \cdot x_{(1,2)} \]

linear!

- Group everything:

\[
\begin{bmatrix}
\tau_{(1,2)} \\
\tau_{(1,3)} \\
\tau_{(1,4)} \\
\vdots \\
\tau_{(M-1,M)}
\end{bmatrix} = 
\begin{bmatrix}
x_{(1,2)}^T \\
x_{(1,3)}^T \\
x_{(1,4)}^T \\
\vdots \\
x_{(M-1,M)}^T
\end{bmatrix} k \iff \tau = Xk
\]
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DOA estimation - planar waves

- To find out where the source is, invert:
  \[ \hat{k} = \left( X^T X \right)^{-1} X^T \hat{\tau} \]

- Propagation speed not needed at all
  \[ \leftarrow \text{Solution gives an estimate } (\|k\| = \frac{1}{c}) \]

- A linear solution, no compensation for nonlinearities required

- Lower bound for error:
  \[ |c\|\hat{k} - 1| \]
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Problems

• Time delays may be erroneous
  – Quantization
  – Difficult signals (noise, reverb)
  – Hardware problems

• Array configuration may be unknown

• Speech applications require accuracy and fast updates
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## Solutions

<table>
<thead>
<tr>
<th>Closed-form solutions</th>
<th>Grid-based searches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least squares</td>
<td>MAD</td>
</tr>
<tr>
<td>Averaging with subarrays</td>
<td>Count-distance</td>
</tr>
<tr>
<td>Fast, but not very robust</td>
<td>More robust, but slow</td>
</tr>
<tr>
<td>Work well with small errors</td>
<td>Can avoid a few large errors</td>
</tr>
</tbody>
</table>
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Time delay selection

- Sum of time delays on a closed path should be zero:

\[ \sum_{(i,j) \in P} x_{(i,j)} = 0 \rightarrow \sum_{(i,j) \in P} x_{(i,j)}^T k = 0 \]

\[ \rightarrow \sum_{(i,j) \in P} \tau_{(i,j)} = 0 \]

- Test all triangles

\[ \hat{\tau}_{(i,j)} + \hat{\tau}_{(j,m)} + \hat{\tau}_{(m,i)} < \text{THR} \]

- Confidence of \( \tau_{(i,j)} \) is the number of tests passed by triangles including \( \tau_{(i,j)} \)

- DOA from a 3-D set of time delays with maximal minimum confidence
Failure detection

- Channel failure
  \[\rightarrow\] all time delays from channel have zero confidence

- Detect with a buffer

- Remove detected channels from processing

- Experiment: insert failures and attempt detection
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## Results

- Confusion matrix for number of failures

<table>
<thead>
<tr>
<th>Number of detected failures</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>98.9</td>
<td>1.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>88.0</td>
<td>10.2</td>
<td>1.2</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>53.3</td>
<td>20.3</td>
<td>11.8</td>
<td>14.6</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>6.1</td>
<td>13.8</td>
<td>80.1</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
<td>99.7</td>
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<tr>
<td>8</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>99.9</td>
</tr>
</tbody>
</table>
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Results

- Were correct sensors detected?

<table>
<thead>
<tr>
<th></th>
<th>correct order % of all</th>
<th>correct order % of correct #</th>
<th>correct set % of all</th>
<th>correct set % of correct #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>98.9</td>
<td>100.0</td>
<td>98.9</td>
<td>100.0</td>
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<tr>
<td>4</td>
<td>87.9</td>
<td>100.0</td>
<td>88.0</td>
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<tr>
<td>5</td>
<td>52.9</td>
<td>99.3</td>
<td>53.3</td>
<td>100.0</td>
</tr>
<tr>
<td>6</td>
<td>5.9</td>
<td>96.6</td>
<td>6.1</td>
<td>100.0</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
<td>75.0</td>
<td>0.3</td>
<td>91.7</td>
</tr>
<tr>
<td>8</td>
<td>36.3</td>
<td>36.3</td>
<td>99.9</td>
<td>100.0</td>
</tr>
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</table>
DOA estimation performance

<table>
<thead>
<tr>
<th>Method</th>
<th>0</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>LS</td>
<td>2.9</td>
<td>38.5</td>
<td>56.1</td>
<td>65.8</td>
<td>70.8</td>
<td>74.9</td>
</tr>
<tr>
<td>LSD</td>
<td>2.9</td>
<td>4.1</td>
<td>5.3</td>
<td>6.9</td>
<td>9.4</td>
<td>13.2</td>
</tr>
<tr>
<td>AVG</td>
<td>3.1</td>
<td>38.3</td>
<td>55.9</td>
<td>65.6</td>
<td>70.7</td>
<td>74.8</td>
</tr>
<tr>
<td>AVGD</td>
<td>3.1</td>
<td>4.2</td>
<td>5.4</td>
<td>7.0</td>
<td>9.4</td>
<td>13.2</td>
</tr>
<tr>
<td>TDS</td>
<td>7.4</td>
<td>7.9</td>
<td>9.8</td>
<td>14.3</td>
<td>21.5</td>
<td>30.8</td>
</tr>
<tr>
<td>TDSD</td>
<td>7.4</td>
<td>7.7</td>
<td>8.2</td>
<td>9.0</td>
<td>10.1</td>
<td>11.6</td>
</tr>
<tr>
<td>CNT</td>
<td>3.0</td>
<td>20.6</td>
<td>33.2</td>
<td>43.0</td>
<td>50.1</td>
<td>55.8</td>
</tr>
<tr>
<td>CNTD</td>
<td>3.0</td>
<td>3.7</td>
<td>4.7</td>
<td>6.0</td>
<td>8.2</td>
<td>10.4</td>
</tr>
</tbody>
</table>
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Results

0 failures

<table>
<thead>
<tr>
<th>Reference</th>
<th>LS</th>
<th>LSD</th>
<th>AVG</th>
<th>AVGD</th>
<th>TDS</th>
<th>TDSD</th>
<th>CNT</th>
<th>CNTD</th>
</tr>
</thead>
</table>

![Graph](image)
Robust direction of arrival estimation

Results

![Graph showing error distribution and failures for different methods: Reference, LS, LSD, AVG, AVGD, TDS, TDSD, CNT, and CNTD. The graph includes a scale for error in degrees and a y-axis ranging from 0 to 1, with markers for 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1. Failures are indicated with markers along the x-axis.](image-url)
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Results

![Graph showing results for different methods: Reference, LS, LSD, AVG, AVGD, TDS, TDSD, CNT, CNTD. The graph displays error ratios against error in degrees. There are two failures noted at a certain error ratio.](image)

- Reference
- LS
- LSD
- AVG
- AVGD
- TDS
- TDSD
- CNT
- CNTD
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Results

![Graph showing error ratio with degrees on the x-axis and ratio on the y-axis. The graph includes lines for Reference, LS, LSD, AVG, AVGD, TDS, TDSD, CNT, and CNTD. There are 3 failures indicated.]
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Results

![Graph showing results for different methods]
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Results

![Graph showing error vs time for different detection and failure points. The graph includes markers for 'LS' and 'LSD' with distinct markers for first and second detections and failures.](image-url)
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Example: performance analysis

- Cumulative distribution of errors (simulated)

- 3 and 4 sensors, 0.5 m tetrahedron/triangle array, sampling rate 48 kHz
Example: effect of propagation speed

- Angular RMS error as a function of true propagation speed

[Diagram showing RMS error vs. propagation speed with three lines representing different array configurations: 3D-array, 4 sensors (reference), 2D-array, 4 sensors, 2D-array, 3 sensors.]

- Same setup as in previous example, assumed propagation speed 343 m/s
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Example: error detection with propagation speed
Conclusions

- Time delay based methods are quite robust
- Good accuracy, low complexity
- Confidence factors will increase robustness
- For good & fast results, use LSD
- Propagation speed not needed in estimation
  → Can be used for error detection and minimization
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Future directions

- Optimum array configurations
- Activity detection
- Faster grid-based methods
- Automatic selection of method
- Time delay errors vs. array errors, automatic correction
- Multiple sources
- Data needed
Acknowledgements

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- Adam & Chuck
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  - Atte Virtanen
<table>
<thead>
<tr>
<th>Thanks for the cash</th>
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- Tampere University of Technology
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- Finnish Foundation for Advancement of Technology (TES)
- The Emil Aaltonen Foundation
The Sapphire Challenge

- Find some published work about time delay based propagation vector estimation


- Win a bottle of Bombay Sapphire!

Please drink responsibly