Hebb’s Rule (1949)

If unit $i$ repeatedly participates in the firing of unit $j$ then the weight from $i$ to $j$ increases.

problems: saturation, lack of specificity

partial solution: decay

anti-hebbian learning (Stent, 1973):

The weight from $i$ to $j$ decreases if unit $j$ fires, but unit $i$ does not.

another variant:

The weight from $i$ to $j$ decreases if unit $i$ fires, but unit $j$ does not.
A possible interpretation of Hebb’s rule

\begin{equation}
    w_{ij} = \frac{\text{the number of times both } i \text{ and } j \text{ fire}}{\text{the number of times } j \text{ fires}}
\end{equation}

How often when unit $j$ was firing, was unit $i$ also firing?
BCM rule (Bienenstock, Cooper, and Munro, 1982)

\[ o_j = \text{net}_j = \sum_i w_{ij} \cdot o_i \]

\[ \Delta w_{ij}(t) = \Phi(o_j(t)) \cdot o_i(t) - \epsilon \cdot w_{ij}(t) \]

\( \Phi \) is a scalar function such that:

\[ \text{sign}(\Phi(o_j(t))) = \text{sign}(o_j(t) - \theta_M) \]

Weight vector is driven in the direction of the input vector if the output is large (above \( \theta_M \)), or opposite to the direction of the input vector if the output is small (below \( \theta_M \)).
BCM rule ...variable threshold, $\theta_M$

$$o_j = net_j = \sum_i w_{ij}.o_i$$

$$\Delta w_{ij}(t) = \Phi(o_j(t), \sigma_j(t)).o_i(t) - \epsilon.w_{ij}(t)$$

$\Phi$ is now a function of $o_j(t)$ as well as $\sigma_j(t)$, the average output.

as before

$$\text{sign}(\Phi(o_j(t)), \sigma_j(t)) = \text{sign}(o_j(t) - \theta_M)$$

but

$$\theta_M(t) = \left(\frac{\sigma_j(t)}{c}\right)^p . \sigma_j(t)$$

$$\Phi(0, \sigma_j) = 0 \text{ for all } \sigma_j$$

where $c$ and $p$ are constants
Output of a node = input vector \cdot weight vector

If weight and input vectors are normalized

\[ output = \cos \alpha \]

where \( \alpha \) is the angle between the weight and input vectors

Learning algorithm:

1. Present an input pattern to the net
2. Identify the node with the highest response
3. Rotate its weight vector toward the input vector.
4. Repeat

(Spatial competition, cf. temporal competition of BCM)