Fair and Robust Power Allocation Rules for Multiple Access Channels

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***Abstract*— We introduce some game theoretic notions of fairness and robustness in determining power allocations for the multiple access channel. We describe a procedure, the wireless fair-share protocol (WFS) which satisfies a strong fairness requirement and in the case where devices have "elastic demands" leads to games with unique and non-manipulable Nash equilibria. When there are multiple channels, the Nash equilibrium are also efficient in the allocation of power among the channels.**

***Index Terms*— Multi-access channel, power allocation, fairness, game theory.**

I. Introduction

It is well known that in the multiple access channel there is a large feasible region. In this paper we consider the problem of how to choose among these possible power allocations. Our analysis is motivated by settings in which a large number of heterogeneous wireless devices (implicitly) choose an allocation, perhaps even without a detailed protocol.

The first part of our analysis considers the fairness of an allocation given a fixed set of data rate requirements. We discuss the idea of a "stand-alone" bound, which provides basic service guarantees to a device independent of the requirements for the other devices. This would allow the designer of a new device to "budget" for worst case performance. We show that both max-min and proportional fairness do not satisfy this requirement but the wireless fair-share rule (WFS) does.

The second part of our analysis considers adaptive devices which have "elastic demand". For example, consider laptops with wifi connections. Depending on the use – websurfing, file transfer or online chat – laptops might have different preferences, such as when deciding between low power, but low data rate and higher power, with a higher data rate. Similarly for devices in a sensor net. Given an allocation rule, for fixed requirements, the devices would then choose the data rate that maximizes their "utility" which would be a function of both their data rate and power requirement. However, this choice would depend on the choices of the other devices, leading to a noncooperative game. Thus, we would like to design an allocation rule such that this game has "nice" properties.

One basic requirement is that the Nash equilibrium of this game should exist and be unique. As we will see, neither max-min nor proportional fairness satisfy this requirement; however, WFS does. In fact games with WFS also have the desirable property that they are difficult to manipulate, either by a single sophisticated device or group of devices.

Lastly we show that these results extend to the case with multiple channels. In addition, in equilibrium, the power is allocated fairly among these channels.

A. Related Work

This power allocation problem has recently been studied in [1] using the tools of cooperative game theory and complementary approach to ours, since their model considers the complexity of self-enforcing contracts among devices, while ours posits much simpler interactions. While their model applies mainly to settings with a stable set of (game theoretically) sophisticated devices, ours applies more to settings where the set of devices varies and the devices do not exert much computational effort on strategic issues. In addition, our notions of fairness differ significantly from theirs.

II. Model and Fairness

Consider the standard multiple access channel with n devices $i \in \{1, \dots, n\} = S$. Given a set of transmission rates r_i we are interested in finding a set of power requirements, p_i, such that (r, p) is feasible.[1]

It is well known that an allocation is feasible if

$$|r_i|_{S'} \leq C(|p|_{S'}) \quad \forall S' \subseteq S,$$

where $C(x) = log(1 + x/N)/2$ is the channel capacity for a Gaussian channel [2], N is the noise level and for any vector, x, $|x|_{S'} = \sum_{i \in S'} x_i$ for any $S' \subseteq S$, $|x| = |x|_S$ and $|x|_{-i} = |x|_{S \setminus i}$. (Note that we use natural logarithms for analytic simplicity.) It is convenient to use the inverse of the channel capacity function $c(y) = N(e^y - 1)$ in which case the feasibility constraints can be rewritten as

$$|p|_{S'} \geq c(|r_i|_{S'}) \quad \forall S' \subseteq S.$$

Note that the inverse channel capacity function is convex and strictly increasing.

We define an allocation rule to be a mapping $\mathcal{F} : r \rightarrow p$ where $\mathcal{F}(r)_i$ is the power requirement for device i when the vector of requirements is r.

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[1] See [2] for a formal definition of feasibility.

A. Max-min and Proportional Fairness

Two well known allocation rules are max-min and proportional fairness. The max-min allocation rule, $\mathcal{F}^{mm}$, solves:

$$\{\max_p \min_i p_i \mid |p|_{S'} \geq c(|r_i|_{S'}) \ \ \forall S' \subseteq S\},$$

while the proportional allocation rule $\mathcal{F}^{pr}$ solves:

$$\{\max_{p_i} \min_i p_i/r_i \mid |p|_{S'} \geq c(|r_i|_{S'}) \ \ \forall S' \subseteq S\}.$$

In our setting, the proportional rule is simply given by

$$\mathcal{F}^{pr}(r)_i = \frac{r_i c(|r|)}{|r|},$$

where all devices get the same value of p_i/r_i.

Theorem 1: For any r, $\mathcal{F}^{pr}(r)$ is feasible.

Proof: This follows from the convexity of $c(\cdot)$. Feasibility requires that for all $S' \subseteq S$

$$|\mathcal{F}^{pr}(r)|_{S'} \geq c(|r_i|_{S'}).$$

From the formula for $\mathcal{F}^{pr}(\cdot)$ we see that

$$|\mathcal{F}^{pr}(r)|_{S'} = |r|_{S'} \frac{c(|r|)}{|r|} \geq |r|_{S'} \frac{c(|r|_{S'})}{|r|_{S'}} = c(|r_i|_{S'})$$

where the inequality arises from fact that $c(\cdot)$ is convex, increasing and $c(0) = 0$. ∎

For the max-min rule there are cases where devices get different values of p_i and the formula is not as simple; however in examples it is useful to note that when there are two devices and $r_1, r_2 > \log(2)$ then both devices do get the same $p_i = c(|r|)/2$.

B. Protectiveness

First, we note that both max-min and proportional fairness are problematic in this setting. Consider an example with 2 devices where $\log(2) < r_1 << r_2$ under max-min fairness. Then $p_i = \mathcal{F}^{mm}(r)_i = c(r_1 + r_2)/2$. Note that p_1^m is strictly increasing in r_2 and becomes arbitrarily large for large values of r_2. This is also true for proportional fairness.

This seems unfair to device 1 which is paying a large price for device 2's greediness. This motivates the following definition.

Definition 1: An allocation rule is protective with stand-alone-function f if for any r, i, $\mathcal{F}_i(r) \leq f(r_i, n)$. An allocation rule is fair if there exists some stand-alone-function for which it is fair.

From the example above we have the following:

Theorem 2: Neither $\mathcal{F}^{mm}$ nor $\mathcal{F}^{pr}$ are protective.

Of course, we are interested in fair allocation rules with "small" stand alone functions. Clearly, by symmetry, the best possible stand-alone-function is given by $f(r_i) = c(nr_i)/n$. We will now show an allocation rule which satisfies this best possible stand alone function.

III. Wireless Fair-Share (WFS)

It is interesting to note that the feasibility conditions for our problem are structurally equivalent to those for a multi-class m/m/1 queue; simply let $c(x) = (\mu - x)^{-1}$ and interpret p_i as the average number of class i jobs, r_i as the rate of class i jobs, where μ is the capacity of the queue [3].

In [3], Shenker proposed the fair-share queuing formula for such queues and we now show that one can apply the same ideas in the wireless setting.

In the wireless setting, we can define the fair-share allocation as follows: given a vector of rates, r compute the vector p as follows. Reorder the devices so that $r_1 \leq r_2 \leq \cdots \leq r_n$. Now the fair-share allocation computes the allocation as follows. First compute

$$q_i = [C(\sum_{j=1}^{i-1} r_j + (n-i+1)r_i) - \sum_{j=1}^{i-1}(n-j+1)q_j]/(n-i+1)$$

then set

$$p_i = \sum_{j=1}^{i} q_j.$$

We will denote this function as $\mathcal{F}^{fs}(r)$.

For example,

$$p_1 = \mathcal{F}_1^{fs}(r) = c(nr_1)/n,$$

so the power requirement for the device requesting the lowest data rate is independent of the actual requirements of the devices with higher requirements and is precisely that equal share if all devices had data rate r_1. This is actually the defining principle of the serial mechanism, for example $p_2 = [c(r_1 + (n-1)r_2) - p_1]/(n-1)$, which is independent of the specific demands for devices with higher demands.[2]

Using the convexity of $c(\cdot)$ one can show that this allocation is feasible.

Theorem 3 (Shenker 1995): For any r, $\mathcal{F}^{fs}(r)$ is feasible.

Proof: This is proven in [3].

In addition, WFS is protective.

Theorem 4 (Moulin and Shenker, 1992): WFS is protective with the best possible stand-alone-function, $f(r_i) = C(nr_i)/n$.

Proof: This is shown in [4].

This relation has the following interpretation: assume that all devices have the same rate requirement, $r_i = r_j, \forall i, j$. Then it is natural for each device to be allocated the same power requirement,$p_i = p_j$, $\forall i, j$. This implies that $p_i = C(nr_i)/n$. Thus, we see that WFS guarantees that no device ever requires a higher p_i than would be demanded if all devices had identical demands. In fact, all devices, except the one with the smallest r_i, require strictly less power.

The following technical characterizations of WFS will be important in our analysis:

Lemma 5 (Shenker 1995): Assume that the devices are ordered so that $r_1 \leq r_2 \leq \cdots \leq r_n$. Then for all i, $\mathcal{F}^{fs}(r)_i$

[2] One can use these ideas to extend the fair share mechanism to other wireless settings, such as TDMA or FDMA in which cases the allocation rules are naturally fair – an equal split of either time or bandwidth.

is continuously differentiable in r, and $\partial \mathcal{F}(r)_i/\partial r_i$ is strictly increasing in r_j for $j \le i$ and independent of r_j for $j > i$.

We will next show that WFS has other desirable properties not shared by max-min or proportional fairness.

IV. Elastic Demand

In this section we extend our analysis to devices with "elastic demand" [3], [5], that is, devices which can make choices among rate-power pairs. For example, in a congested area, a laptop using a wifi connection with a low battery might prefer a lower rate connection to a high rate one with excessive power requirements, while one with a fully charged battery might prefer the other. As is standard in game theory, we denote the objective function for each device by a utility function. We assume that device i has a concave and continuously differentiable utility function $u_i(r_i, p_i)$ which is increasing in r_i and decreasing in p_i. Given a set of rate-power pairs, we assume that a device would prefer the one with the highest utility.

For example, a device could attempt to maximize the information transferred subject to the remaining battery power as discussed in [6], [7]. Another common utility function is quasilinear in power, $u_i(r_i, p_i) = v_i(r_i) - p_i$, where $v(\cdot)$ is an increasing and concave function. However, we will not restrict ourselves to any specific utility function in this paper.

Given an allocation mechanism, $\mathcal{F}$, we get a set of dependent payoff functions:

$$\pi_i(r) = u_i(r_i, \mathcal{F}(r))$$

which defines a noncooperative game in normal form [8]. A focal outcome of a game is the Nash equilibrium, r^N, which is defined by the selfish optimality condition

$$r_i^N = \text{argmax}_{r_i} \pi_i(r_i, r_{-i}^N)$$

where r_{-i} denotes the vector r with its i'th element removed, so for example,
$(r_i', r_{-i}) = (r_1, r_2, \ldots, r_{i-1}, r_i', r_{i+1}, \ldots, r_n)$. Note that depending on the allocation function, the Nash equilibrium may or may not exist. When it is convex, as for $\mathcal{F}^{pr}$ and $\mathcal{F}^{fs}$ existence is guaranteed since the game is convex; however, when it is not convex, such as $\mathcal{F}^{mm}$ there is no guarantee that the equilibrium even exists.

A. Example

Consider a game with 2 devices and linear utilities, $u_i(r_i, p_i) = \alpha_i r_i - p_i$. For simplicity set $N = 1$ and assume that $\alpha_i > 1$. The first order conditions for a Nash equilibrium are

$$\alpha_i = \frac{\partial \mathcal{F}_i(r)}{\partial r_i}.$$

When $\mathcal{F} = \mathcal{F}^{pr}$, and $\alpha_1 = \alpha_2 = 10$ the Nash equilibrium is $r = (1.35, 1.35)$, with $\pi_1 = \pi_2 = 6.56$; however if $\alpha_1 = 20$ while $\alpha_2 = 10$ the Nash equilibrium becomes $r = (2.55, 0.59)$, with $\pi_1 = 7.55$ and $\pi_2 = 1.75$ and note that device 2 has a significantly lower payoff due to device 1's "greed".

Now consider the same game when $\mathcal{F} = \mathcal{F}^{fs}$. Since in both cases $r_1 \ge r_2$ in equilibrium, we can compute r_2 by noting that $\mathcal{F}_2^{fs}(r) = c(2r_2)/2$, so in both cases $r_2 = 1.15$. Thus, in the first case the Nash equilibrium is $r = (1.15, 1.15)$, with $\pi_1 = \pi_2 = 7.01$, while in the second case the Nash equilibrium becomes $r = (1.84, 1.15)$, with $\pi_1 = 7.01$ and $\pi_2 = 22.40$.

There are two interesting things to note. First, when the devices have the same utility functions, WFS leads to higher utility in equilibrium than proportional fairness. We will not pursue this issue, since it is well discussed in [3]; however, we recall that in equilibrium WFS maximizes utility when the devices are identical, but both max-min and proportional fairness (and most other allocation functions) are inefficient in this case. Second, under proportional fairness the less demanding device suffers due to the more demanding device's "greed", while under WFS the less demanding device is protected and it's outcome is not affected by the more demanding device.

Lastly, note that for linear utility functions the Nash equilibria of $\mathcal{F}^{mm}$ are uninteresting – the device with the highest α_i is the only device with positive demand at equilibrium. This is not true for more general utility functions.

B. Uniqueness of Nash Equilibria

As discussed above, all the allocations considered lead to games with Nash equilibria. However, problems could arise if there are multiple Nash equilibria. In such a case there is no natural outcome for the game.

Moulin and Shenker [4] provides a simple method for checking whether equilibria a unique. Given an allocation function and a demand r, define a di-graph on the set of devices by the following rule: there is a directed link from i to j if $\partial_i \mathcal{F}_j(r) \neq 0$ and no link otherwise. (There are no links from a device to itself.) Call this graph $G(\mathcal{F}, p)$.

Theorem 6 (Moulin and Shenker, 1992): Given an allocation function $\mathcal{F}$ and a point r, if $G(\mathcal{F}, p)$ has a directed cycle then there exists a set of utility functions such that the game defined by $\pi_i(r) = u_i(r_i, \mathcal{F}(r))$ has at least 2 Nash equilibria.

This implies that both the max-min and proportional allocations lead to games with multiple equilibria.

Theorem 7:
1)For $\mathcal{F}^{pr}$ and any $r > 0$ there exists a set of utility functions such that the game defined by $\pi_i(r) = u_i(r_i, \mathcal{F}^{pr}(r))$ has multiple Nash equilibria.
2)For $\mathcal{F}^{mm}$, and any $r > \log(2)$ there exists a set of utility functions such that the game defined by $\pi_i(r) = u_i(r_i, \mathcal{F}^{mm}(r))$ has multiple Nash equilibria.

However, WFS always has a unique solution.

Theorem 8: For the WFS allocation rule, $\mathcal{F}^{fs}$, for any set of utility functions, u_i, the game defined by $\pi_i(r) = u_i(r_i, \mathcal{F}^{fs}(r))$ has a unique Nash equilibrium.

Proof: This is proven in a more general setting in [4].

We will review the key ideas in the proof as they provide a recursive procedure for finding the Nash equilibrium. First, for device i define the unanimity demand to be the value of r_i which maximizes its payoff, $r_i^u = \text{argmax } u_i(r_i, \mathcal{F}^{fs}(r_i\bar{1}))$, where $\bar{1} = (1, 1, 1 \ldots, 1)$. Let $i = \text{argmin}_j r_j$. Then, as shown in [4] device i plays r_i^u in the Nash equilibrium. Next, freeze device i's strategy and repeat the procedure for the remaining players. This process yields the Nash equilibrium and can be used to show that it is unique.

C. Manipulability of Nash Equilibria

Even when a game has a unique Nash equilibrium we still need to worry about the ability of sophisticated players manipulating the outcome.

D. Example

Consider our previous example with $N = 1$, 2 devices and linear utilities, $u_i(r_i, p_i) = \alpha_i r_i - p_i$.

Recall that when $\mathcal{F} = \mathcal{F}^{pr}$, and $\alpha_1 = \alpha_2 = 10$ the Nash equilibrium is $r = (1.35, 1.35)$, with $\pi_1 = \pi_2 = 6.56$, while when $\mathcal{F} = \mathcal{F}^{fs}$ the Nash equilibrium is $r = (1.15, 1.15)$, with $\pi_1 = \pi_2 = 7.01$.

First we note that under proportional fairness there is a very simple form of manipulation, both devices colluding to play the WFS equilibrium $r = (1.15, 1.15)$, which would lead to the WFS payoffs $\pi_1 = \pi_2 = 7.01$ (under proportional fairness). Thus, if they could coordinate, the devices would have a strong incentive not to play the Nash equilibrium; however, under WFS they would have no such incentive.

Similarly, a single device could try to manipulate the outcome. For example if device 1 knew that device 2 was optimizing with respect to its strategy, r_1, then it could manipulate the outcome as follows: given r_1, player 2 will choose r_2 to maximize $u_2(r_1, r_2)$. The optimal solution will satisfy the first order conditions:

$$\alpha_2 = \frac{\partial[r_2(e^{r_1+r_2} - 1)/(r_1 + r_2)]}{\partial r_2}.$$

Call the solution $ST_2(r_1)$. Then device 1 could maximize

$$u_1(r_1, \mathcal{F}_1^{pr}(r_1, ST_2(r_1)).$$

This leads to the "Stackelberg" equilibrium with device 1 being the leader, $r = (2.17, 0.82)$ and $\pi = (8.02, 3.02)$. Note that this manipulation is even better for device 1 than the collusive outcome above, but much worse for device 2.

Lastly, we note that even without explicit manipulations, implicit Stackelberg manipulations can occur due to the timing of updates. See [9] for more details and [10] for simulation results.

E. Results

Formally, let $ST^i(r_i)$ be the Nash equilibrium of the game where device i's strategy is fixed at r_i. Thus, $\forall i \neq j$

$$ST_j^i(r_i) = \text{argmax}_{r_j} \pi_i(r_j, r_i, ST^i(r_i)_{-j}).$$

Then a Stackelberg equilibrium of a game with leader i is given by r satisfying $r_{-i} = ST^i(r_i)$ and

$$r_i = \text{argmax}_{r_i} \pi_i(r_i, ST^i(r_i)).$$

We say that a game is not "individually manipulable" if every Stackelberg equilibrium is also a Nash equilibrium.

Theorem 9:
1)For $\mathcal{F}^{pr}$ and any $r > 0$ and any set of strictly concave utility functions which are strictly decreasing in p_i, the induced game is manipulable by any device.
2)For $\mathcal{F}^{mm}$, and any $r > \log(2)$ and any set of strictly concave utility functions which are strictly decreasing in p_i, the induced game is manipulable by any device.

Proof: This follows from [4].

Note that the above theorem applies to any utility function that is strictly decreasing in p_i. However, WFS is not individually manipulable for any set of utility functions.

Theorem 10: For the WFS allocation rule, $\mathcal{F}^{fs}$, for any set of utility functions, u_i, the Nash equilibrium of the induced game is never manipulable by any device.

Proof: This is proven in [4]

Analogously to the previous result, one can show that WFS cannot be manipulated by groups of devices, while max-min and proportional fairness can. (Technically, the equilibrium under WFS is "strong". See [4] for details.)

Lastly, we note that WFS is dynamically stable and a wide variety of distributed optimization mechanisms converge to the Nash equilibrium [9].

V. Multi-Channel Systems

We next consider the case when there are multiple communication channels. For example, some subgroup of the devices have access to a special channel while all devices have access to a shared channel, such as might occur on the unlicensed bands when some devices also have access to a licensed band, such as a cell phone which also runs 802.11.

Number the channels $1, 2, \ldots, n$ and let $S(k)$ be the set of devices that have access to channel k. For each device i let r_i^k be the rate at which it is transmitting on channel k and p_i^k be the power it is using on channel k. By assumption, if $i \notin S(k)$ then $r_i^k = 0$ and $p_i^k = 0$.

Since data rates and power are additive the utility for a device is given by

$$u_i(r_i, p_i) = u_i(\sum_k r_i^k, \sum_k p_i^k)$$

where $u_i(\cdot, \cdot)$ is as in the single channel case. Assuming that all channels use the same allocation rule $p^k = \mathcal{F}(r^k)$ so

$$\pi_i(r_i) = u_i(\sum_k r_i^k, \sum_k \mathcal{F}_i(r^k))$$

which defines the "multichannel game".

We will now show that all of our single channel results for WFS hold in the multichannel case. (Clearly the negative results for max-min and proportional fairness also hold, since the single channel game is a special case of the multi-channel game.)

First we note that a Nash equilibrium of the multi-channel game preserves ordering:

Lemma 11: Let r be a Nash equilibrium of the WFS multichannel game. Then if for some i, j, k $r_i^k \le r_j^k$ then for all k' $r_i^{k'} \le r_j^{k'}$

Proof: Note that the first order conditions for a Nash equilibrium can be written as

$$\partial_{r_i^k}\mathcal{F}_i(r^k) = -\frac{\partial_r u_i(\sum_k r_i^k, \sum_k \mathcal{F}_i(r^k))}{\partial_p u_i(\sum_k r_i^k, \sum_k \mathcal{F}_i(r^k))}$$

for all i, k such that $k \in S(i)$ and $r_i^k > 0$. (Note that ∂_r, ∂_p denote the partial derivatives of $u(\cdot,\cdot)$ with respect to its first and second arguments, respectively.) Thus, in equilibrium for any pair of nonzero rates r_i^k, r_i^m the "marginal costs" must be the same. Now, suppose that there is an equilibrium with $r_i^k < r_j^k$ and $r_i^m > r_j^m$. This would violate Lemma 5 since $\partial_{r_i^k}\mathcal{F}_i(r^k)$ preserves the orderings. ■

Using this lemma, one can prove the following theorem using a slight modification of the analysis in [4].

Theorem 12: The WFS multi-channel game has a unique Nash equilibrium.

In addition, that same proof also shows the following.

Theorem 13: The WFS multi-channel game is never manipulable by any device.

As before it is also true that the multi-channel is not manipulable by groups of devices and has robust convergence properties.

VI. Water Filling

A. Example: 2 channels

Consider an example with 2 channels $N^1 = 1,\ N^2 = 2$, 2 devices and linear utilities, $U_i(r_i^1 + r_i^2, p_i^1 + p_i^2) = \alpha_i(r_i^1 + r_i^2) - (p_i^1 + p_i^2)$ where $\alpha_1 = 5$, $\alpha_2 = 4$. Note that the utility functions are separable as

$$U_i(r_i^1 + r_i^2, p_i^1 + p_i^2) = [\alpha_i r_i^1 - p_i] + [\alpha_i r_i^2 - p_i^2]$$

so the optimization over each channel can be solved separately.

For the case of $\mathcal{F}^{pr}$, we get $r^1 = (1.18, 0.64)$ and $r^2 = (0.76, 0.26)$. From these we see that $p^1 = 5.16$ and $p^2 = 3.53$.

However, for the case of WFS we get $r^1 = (0.92, 0.69)$ and $r^2 = (0.57, 0.35)$ with $p^1 = 3.00$ and $p^2 = 2.00$. Note that

$$p^1 + N^1 = p^2 + N^2,$$

which is precisely the "water filling" condition for the efficient allocation of power between the two channels. As we will see, Nash equilibria of WFS allocate efficiently between channels, while max-min and proportional fairness do not. (As can be seen for proportional fairness in this example and can be checked for max-min fairness.)

B. General Theory

In this section we will assume that all devices have access to all channels,i.e., $S(k) = S$ for all channels k. If this were not true then allocative efficiency might not occur, such as in the case when each device has its own channel. However, when this is true, then the Nash equilibria of WFS are always allocatively efficient.

Theorem 14: In the multi-channel model with $S(k) = S$ for all k, the Nash equilibrium will always be "allocatively efficient" in the sense that allocation of power between different channels is efficient.

Proof: From Lemma 11 we know that there is a "largest" device, j, such that $r_j^k \geq r_i^k$ for all devices i and channels k. Thus, this device is faces a cost function $\mathcal{F}_j^{fs}(r^k) = c(|r^k|) - d^k(r_{-j}^k)$ where $d^k(\cdot)$ is a function that does not depend on r_j. Thus device j's strategy solves:

$$\max_{r_j} u_j(\sum_k r_j^k, \sum_k [c(|r^k|) - d^k(r_{-j}^k)]).$$

This is equivalent to

$$\max_{\lambda} \left\{ \max_{r_j} u_j(\lambda, \sum_k [c(|r^k|) - d^k(r_{-j}^k)]) | \sum_k r_j^k = \lambda \right\}.$$

Since $u_j(\cdot,\cdot)$ is decreasing in its second argument, the inner optimization is equivalent to

$$\min_{r_j} \sum_k [c(|r^k|) - d^k(r_{-j}^k)] | \sum_k r_j^k = \lambda,$$

but since the second term in the sum does not depend on r_j, this is equivalent to

$$\min_{r_j} \sum_k c(|r^k|) | \sum_k r_j^k = \lambda,$$

and thus device j's Nash equilibrium strategy also leads to an allocatively efficient outcome. ■

VII. Concluding Comments

We have described one attractive solution to the problem of choosing a particular feasible allocation. Our solution, WFS, provides performance guarantees, leads to games with unique Nash equilibria and allocates power efficiently among channels. One important open question concerns the complexity of its implementation. One possible avenue is to take advantage of the polymatroidal structure of the problem via a multiple timescale approach as proposes in [11].

Nonetheless, we believe that it is important to understands the (either explicit or implicit) choices in power allocation among devices and think that this issue deserves further study.

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